Dynamics of life models

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Abstract. This is an introduction to the mathematics of life models, with emphasis on the modeling and understanding of the world population.

Preface

You might be perhaps too young for this, as a fresh student now entering college, but have you ever lived with someone, or considered doing so? One question which appears, in every single couple on this planet, is how to share money. And scientifically, this is a very interesting question, in our jargon being called "2 body problem in economics".

Generally speaking, 2 body problems appear all across science, and are usually quite easy to solve, at least with our modern mathematics technology. Of course, all this took some time to be clarified, and with the first such discovery, which actually took a fair amount of time, and work from mankind, over many centuries, being the solution, by Kepler and then Newton, of the 2 body problem in classical mechanics.

In fact, for the story, Newton invented calculus and modern mathematics precisely for solving this problem, and afterwards everyone benefited from this, calculus and mathematics, in order to solve various other 2 body problems, appearing across science.

Remarkably, however, this 2 body problem is not really solved in two important branches of science, namely quantum mechanics, and economics. With quantum mechanics, the problem is that of fully understanding the functioning of the hydrogen atom, which is something not done yet, despite about a century of hard work, since the 1920s. As for economics, well, the 2 body problem here is the one evoked above, husband and wife bitterly arguing, from time to time, about common money. And with this problem being of course not solved yet, despite millenia of studies on it, by mankind.

In short, welcome to Planet Earth, life, human beings and economics, which are, obviously, as difficult as quantum mechanics, and perhaps even more. That is, go find some equilibrium for small or big cities, businesses, states, civilizations and so on, when such an equilibrium is obviously impossible in the context of a 2-person affair.

We will be discussing in this book such questions, with the main aim being that, besides getting introduced to general game theory and life models, all good knowledge to be learned from here, understanding the dynamics of the world population.

Regarding this latter world population, you surely know about it, we are presently 8 billion and counting, which is probably 1000 times what this planet can really afford. The

reasons for this come from the self-regulation of the world population, which naturally appears via famine, disease and war. In recent times, however, due to various reasons, mostly civilizational and religious, but let us not forget to blame science too, or at least certain branches of science, all these 3 levers have been severly weakened. And here we are, in the present world that we know well, basically all overcrowded cities, most of the landscape being reserved for agriculture, animal species dissapearing one by one, land, water and air all polluted, and cherry on the cake, with the whole planet warming.

Do you have a solution to this? I bet you don't, or at least there is no known, wise solution to this. However, as scientists, we can at least try to get familiar with the problem, that is, talking about it, and studying it a bit. We will do this here in this book, after learning the basics, by mathematically studying all sorts of advanced life models, taking into acount the above three main levers, namely famine, disease and war.

Many thanks to my scientific colleagues, for discussions about this, in fact the situation with the world population is a bit similar to that with the electric computers, or internet, these have been since ages in our labs, and came to mainstream only recently. By the way, speaking coming mainstream, thanks to Generation Z, and their Swedish guru, although these don't seem to make a link between global warming, and global population, too bad. Guess that finding will be reserved for Generation Alpha. Finally, many thanks to my cats, good job that they do, global warming certainly does not come from mice.

Cergy, December 2024
Teo Banica

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Or am I caught in a hit-and-run

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Congratulations for having read this book, and no exercises for this final chapter.

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