

Discrete random variables

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ABSTRACT. This is an introduction to probability and discrete random variables. We first discuss binomials, factorials, and the other basic tools of counting, all explained with numerous illustrations. Then we get into the theory of discrete random variables, with the basic theory explained, accompanied by basic illustrations, notably with results about the binomial laws, and with the Poisson laws discussed too. We then discuss some applications of this, to various basic questions in science, and with a look into more advanced methods, of statistics, too. Finally, we come back with mathematics, and discuss a number of more advanced aspects, notably with the theory of the compound Poisson laws, and with a discussion regarding the connections with combinatorics.

Preface

This is an introduction to probability and discrete random variables. We first discuss binomials, factorials, and the other basic tools of counting, all explained with numerous illustrations. Then we get into the theory of discrete random variables, with the basic theory expained, accompanied by basic illustrations, notably with results about the binomial laws, and with the Poisson laws discussed too. We then discuss some applications of this, to various basic questions in science, and with a look into more advanced methods, of statistics, too. Finally, we come back with mathematics, and discuss a number of more advanced aspects, notably with the theory of the compound Poisson laws, and with a discussion regarding the connections with combinatorics.

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Part I

Numbers, counting

CHAPTER 1

1a.

1b.

1c.

1d.

1e. Exercises

Exercises:

EXERCISE 1.1.

EXERCISE 1.2.

EXERCISE 1.3.

EXERCISE 1.4.

EXERCISE 1.5.

EXERCISE 1.6.

EXERCISE 1.7.

EXERCISE 1.8.

Bonus exercise.

CHAPTER 2

2a.

2b.

2c.

2d.

2e. Exercises

Exercises:

EXERCISE 2.1.

EXERCISE 2.2.

EXERCISE 2.3.

EXERCISE 2.4.

EXERCISE 2.5.

EXERCISE 2.6.

EXERCISE 2.7.

EXERCISE 2.8.

Bonus exercise.

CHAPTER 3

3a.

3b.

3c.

3d.

3e. Exercises

Exercises:

EXERCISE 3.1.

EXERCISE 3.2.

EXERCISE 3.3.

EXERCISE 3.4.

EXERCISE 3.5.

EXERCISE 3.6.

EXERCISE 3.7.

EXERCISE 3.8.

Bonus exercise.

CHAPTER 4

4a.

4b.

4c.

4d.

4e. Exercises

Exercises:

EXERCISE 4.1.

EXERCISE 4.2.

EXERCISE 4.3.

EXERCISE 4.4.

EXERCISE 4.5.

EXERCISE 4.6.

EXERCISE 4.7.

EXERCISE 4.8.

Bonus exercise.

Part II

Random variables

CHAPTER 5

5a.

5b.

5c.

5d.

5e. Exercises

Exercises:

EXERCISE 5.1.

EXERCISE 5.2.

EXERCISE 5.3.

EXERCISE 5.4.

EXERCISE 5.5.

EXERCISE 5.6.

EXERCISE 5.7.

EXERCISE 5.8.

Bonus exercise.

CHAPTER 6

6a.

6b.

6c.

6d.

6e. Exercises

Exercises:

EXERCISE 6.1.

EXERCISE 6.2.

EXERCISE 6.3.

EXERCISE 6.4.

EXERCISE 6.5.

EXERCISE 6.6.

EXERCISE 6.7.

EXERCISE 6.8.

Bonus exercise.

CHAPTER 7

7a.

7b.

7c.

7d.

7e. Exercises

Exercises:

EXERCISE 7.1.

EXERCISE 7.2.

EXERCISE 7.3.

EXERCISE 7.4.

EXERCISE 7.5.

EXERCISE 7.6.

EXERCISE 7.7.

EXERCISE 7.8.

Bonus exercise.

CHAPTER 8

8a.

8b.

8c.

8d.

8e. Exercises

Exercises:

EXERCISE 8.1.

EXERCISE 8.2.

EXERCISE 8.3.

EXERCISE 8.4.

EXERCISE 8.5.

EXERCISE 8.6.

EXERCISE 8.7.

EXERCISE 8.8.

Bonus exercise.

Part III

Life and games

CHAPTER 9

9a.

9b.

9c.

9d.

9e. Exercises

Exercises:

EXERCISE 9.1.

EXERCISE 9.2.

EXERCISE 9.3.

EXERCISE 9.4.

EXERCISE 9.5.

EXERCISE 9.6.

EXERCISE 9.7.

EXERCISE 9.8.

Bonus exercise.

CHAPTER 10

10a.

10b.

10c.

10d.

10e. Exercises

Exercises:

EXERCISE 10.1.

EXERCISE 10.2.

EXERCISE 10.3.

EXERCISE 10.4.

EXERCISE 10.5.

EXERCISE 10.6.

EXERCISE 10.7.

EXERCISE 10.8.

Bonus exercise.

CHAPTER 11

11a.

11b.

11c.

11d.

11e. Exercises

Exercises:

EXERCISE 11.1.

EXERCISE 11.2.

EXERCISE 11.3.

EXERCISE 11.4.

EXERCISE 11.5.

EXERCISE 11.6.

EXERCISE 11.7.

EXERCISE 11.8.

Bonus exercise.

CHAPTER 12

12a.

12b.

12c.

12d.

12e. Exercises

Exercises:

EXERCISE 12.1.

EXERCISE 12.2.

EXERCISE 12.3.

EXERCISE 12.4.

EXERCISE 12.5.

EXERCISE 12.6.

EXERCISE 12.7.

EXERCISE 12.8.

Bonus exercise.

Part IV

Advanced aspects

CHAPTER 13

13a.

13b.

13c.

13d.

13e. Exercises

Exercises:

EXERCISE 13.1.

EXERCISE 13.2.

EXERCISE 13.3.

EXERCISE 13.4.

EXERCISE 13.5.

EXERCISE 13.6.

EXERCISE 13.7.

EXERCISE 13.8.

Bonus exercise.

CHAPTER 14

14a.

14b.

14c.

14d.

14e. Exercises

Exercises:

EXERCISE 14.1.

EXERCISE 14.2.

EXERCISE 14.3.

EXERCISE 14.4.

EXERCISE 14.5.

EXERCISE 14.6.

EXERCISE 14.7.

EXERCISE 14.8.

Bonus exercise.

CHAPTER 15

15a.

15b.

15c.

15d.

15e. Exercises

Exercises:

EXERCISE 15.1.

EXERCISE 15.2.

EXERCISE 15.3.

EXERCISE 15.4.

EXERCISE 15.5.

EXERCISE 15.6.

EXERCISE 15.7.

EXERCISE 15.8.

Bonus exercise.

CHAPTER 16

16a.

16b.

16c.

16d.

16e. Exercises

Congratulations for having read this book, and no exercises for this final chapter.

Bibliography

- [1] V.I. Arnold, *Mathematical methods of classical mechanics*, Springer (1974).
- [2] V.I. Arnold, *Lectures on partial differential equations*, Springer (1997).
- [3] M.F. Atiyah, *The geometry and physics of knots*, Cambridge Univ. Press (1990).
- [4] T. Banica, *Calculus and applications* (2024).
- [5] T. Banica, *Introduction to modern physics* (2025).
- [6] R.J. Baxter, *Exactly solved models in statistical mechanics*, Academic Press (1982).
- [7] S.M. Carroll, *Spacetime and geometry*, Cambridge Univ. Press (2004).
- [8] D.D. Clayton, *Principles of stellar evolution and nucleosynthesis*, Univ. of Chicago Press (1968).
- [9] A. Connes, *Noncommutative geometry*, Academic Press (1994).
- [10] W.N. Cottingham and D.A. Greenwood, *An introduction to the standard model of particle physics*, Cambridge Univ. Press (2012).
- [11] P.A. Davidson, *Introduction to magnetohydrodynamics*, Cambridge Univ. Press (2001).
- [12] P. Di Francesco, P. Mathieu and D. Sénéchal, *Conformal field theory*, Springer (1996).
- [13] P.A.M. Dirac, *Principles of quantum mechanics*, Oxford Univ. Press (1930).
- [14] S. Dodelson, *Modern cosmology*, Academic Press (2003).
- [15] A. Einstein, *Relativity: the special and the general theory*, Dover (1916).
- [16] L.C. Evans, *Partial differential equations*, AMS (1998).
- [17] L.D. Faddeev and L. A. Takhtajan, *Hamiltonian methods in the theory of solitons*, Springer (2007).
- [18] E. Fermi, *Thermodynamics*, Dover (1937).
- [19] R.P. Feynman, R.B. Leighton and M. Sands, *The Feynman lectures on physics I: mainly mechanics, radiation and heat*, Caltech (1963).
- [20] R.P. Feynman, R.B. Leighton and M. Sands, *The Feynman lectures on physics II: mainly electromagnetism and matter*, Caltech (1964).
- [21] R.P. Feynman, R.B. Leighton and M. Sands, *The Feynman lectures on physics III: quantum mechanics*, Caltech (1966).
- [22] R.P. Feynman and A.R. Hibbs, *Quantum mechanics and path integrals*, Dover (1965).
- [23] A.P. French, *Special relativity*, Taylor and Francis (1968).
- [24] N. Goldenfeld, *Lectures on phase transitions and the renormalization group*, CRC Press (1992).

- [25] H. Goldstein, C. Safko and J. Poole, *Classical mechanics*, Addison-Wesley (1980).
- [26] M.B. Green, J.H. Schwarz and E. Witten, *Superstring theory*, Cambridge Univ. Press (2012).
- [27] D.J. Griffiths, *Introduction to electrodynamics*, Cambridge Univ. Press (2017).
- [28] D.J. Griffiths and D.F. Schroeter, *Introduction to quantum mechanics*, Cambridge Univ. Press (2018).
- [29] D.J. Griffiths, *Introduction to elementary particles*, Wiley (2020).
- [30] K. Huang, *Quantum field theory*, Wiley (1998).
- [31] K. Huang, *Quarks, leptons and gauge fields*, World Scientific (1982).
- [32] C. Itzykson and J.B. Zuber, *Quantum field theory*, Dover (1980).
- [33] L.P. Kadanoff, *Statistical physics: statics, dynamics and renormalization*, World Scientific (2000).
- [34] T. Kibble and F.H. Berkshire, *Classical mechanics*, Imperial College Press (1966).
- [35] M. Kumar, *Quantum: Einstein, Bohr, and the great debate about the nature of reality*, Norton (2009).
- [36] T. Lancaster and K.M. Blundell, *Quantum field theory for the gifted amateur*, Oxford Univ. Press (2014).
- [37] L.D. Landau and E.M. Lifshitz, *Mechanics*, Pergamon Press (1960).
- [38] L.D. Landau and E.M. Lifshitz, *The classical theory of fields*, Addison-Wesley (1951).
- [39] L.D. Landau and E.M. Lifshitz, *Quantum mechanics: non-relativistic theory*, Pergamon Press (1959).
- [40] V.B. Berestetskii, E.M. Lifshitz and L.P. Pitaevskii, *Quantum electrodynamics*, Butterworth-Heinemann (1982).
- [41] R.K. Pathria and P.D. Beale, *Statistical mechanics*, Elsevier (1972).
- [42] M. Peskin and D.V. Schroeder, *An introduction to quantum field theory*, CRC Press (1995).
- [43] M. Schwartz, *Principles of electrodynamics*, Dover (1972).
- [44] J. Schwinger, L.L. DeRaad Jr., K.A. Milton and W.Y. Tsai, *Classical electrodynamics*, CRC Press (1998).
- [45] J. Schwinger and B.H. Englert, *Quantum mechanics: symbolism of atomic measurements*, Springer (2001).
- [46] J.R. Taylor, *Classical mechanics*, Univ. Science Books (2003).
- [47] J. von Neumann, *Mathematical foundations of quantum mechanics*, Princeton Univ. Press (1955).
- [48] S. Weinberg, *Foundations of modern physics*, Cambridge Univ. Press (2011).
- [49] S. Weinberg, *Lectures on quantum mechanics*, Cambridge Univ. Press (2012).
- [50] S. Weinberg, *Lectures on astrophysics*, Cambridge Univ. Press (2019).