Probability on groups

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Abstract. This is an introduction to probability on groups.

Preface

This is an introduction to probability on groups.

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Bonus exercise.

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Congratulations for having read this book, and no exercises for this final chapter.

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