Mathematics of brain

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Abstract. This is an introduction to the functioning and mathematics of the human brain. We insist on both biological and mathematical aspects.

Preface

Have you noticed that certain individuals are smarter than some other? Well, this is totally wrong. We all have the same brain, this is how our human species is made, and with this being a scientifically proved fact. And also, as a matter of allowing a bit of room for error, this common brain is actually far bigger than what is needed for activities requiring "smartness", and with this being a scientifically proved fact too. So, definitely no way for the above smartness question, at least if you call yourself a scientist.

What happens, however, is that for various non-intellectual activities, such as doing hard physical work, or high level sports, or fighting with viruses and microbes, or with famine and such, we are definitely not equal. Here it matters a lot how you were born, how your parents and ancestors were too, and so on, and you cannot do much about it. In fact, this is what truly brings the diversity of our human species.

As an illustration here, take Albert Einstein and Usain Bolt. Bolt did sports, but if he were to be interested in physics instead, and as passioned and motivated about it as Einstein, please be sure that he could have done a physics work of similar magnitude to that of Einstein. With this meaning fully elucidating the nature of the weak and strong forces, unifying with gravity and electromagnetism, and computing the fine structure constant too. On the opposite, assuming that Einstein was to be interested in running the 100 meters, and as passioned and dedicated as Bolt about it, I'm not quite sure that he could have done a good work here, say winning the Olympics, at that time.

So, this is one thing to be known, before getting interested in science of any kind, in relation with the brain, or in science at large, we are all equal, brain-wise.

Which, of course, wipes out familiar concepts like "being gifted", which are dear to the present, modern Western civilization. So, if reading these lines, and coming from such a civilization, but still want to be a serious scientist, please have a bit of thinking at all this, and rearrange your thoughts. All this is mandatory, in order to do science.

In fact, and for ending this philosophical discussion, the "being gifted" concept is something very recent, reminiscent of a decaying civilization, where basically no one wants to work hard, and invents all sorts of excuses instead. And I'm saying this as an academic, daily confronted with this problem, there are many nice questions to work

on hard, and not many volunteers around, none being gifted enough, they say. On the opposite, and with a ray of light coming here, showing in particular the fragility of the "being gifted" concept, none of this, or almost, happens in the context of teaching. I mean, at the university we have classes and exams, with good grades being assigned to hard-working students, and bad grades to the lazy ones, and kids ok with this. Nice.

Time perhaps to talk about the present book? We have already learned a lot about the human brain, from the above, but please don't go away, there are 400 more pages to follow, on the same subject, all sorts of interesting scientific technicalities.

The book is organized in 4 parts, with Part I discussing the various animals and their brains, that is, biology material, then Part II and Part III going into the human brain, with more biology, followed by some mathematics, and finally with Part IV discussing all sorts of engineering speculations and feats, inspired from the human brain.

Many thanks to my colleagues, I would say, as I always say at the end of each book preface, but in view of the above, I would rather go with warm thanks to my students, the future is yours, and you are certainly smarter than us. Many thanks as well to my cats, we humans are not equal to cats, brain-wise, this is something that I forgot to mention, in the above discussion, but we will certainly talk about this too, in this book.

Cergy, December 2024
Teo Banica

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