

Principles of metallurgy

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ABSTRACT. This is an introduction to metals and metallurgy, and to early metal engineering too, from the late Stone Age up to the beginnings of the Industrial Age. We first discuss the various metals and their basic mechanical properties, from a modern physics perspective. Then we embark on a detailed metal genesis discussion, featuring stars, planets, chemistry and geology. We discuss then the basics of metallurgy, from the ancient times, and with a few modern aspects too. Finally, we focus on iron and steel, with a lengthy theoretical discussion about both, and what can be done with them.

Preface

The story goes that you can survive in the wild only with a knife. Or at least our modern, cozy breed of homo sapiens is supposed to do so, and this because our Stone Age ancestors were used to survive, or rather pleasantly live, without the said knife.

Of course, this requires some training and skill. Setting up a fire, even with your knife complemented by a match, can be a tricky business in bad weather conditions, and in the hope that you will do some experiments here, and report results better than mine. As for catching wild animals for food, my own experience with my semi-wild cats, caught for hugs instead of food, and of course without knife, is not very conclusive either, and in the hope again that you can do some experiments here, and bon appetit.

Interestingly now, the needed knife can be actually manufactured by yourself in the wild, provided that you are in a favorable geological landscape, have some metallurgy know-how, and managed to find shelter, fire and food for a day or two. And with this being something truly amazing, at the beginnings of our modern civilization, pulling humanity from the Stone Age to the Bronze Age, and then to the present Iron Age.

In short, modern civilization is all about metal. This book stands as an introduction to this, metals and metallurgy, quite often regarded from the perspective of modern physics and chemistry, and to some early metal engineering too, from the late Stone Age up to the beginnings of the Industrial Age. That is, we will try to be modern on certain aspects, and not modern at all on some other, the plan being as follows:

I - We will first discuss the various metals and their basic mechanical properties, from a modern physics perspective. Which is actually something quite tricky, to predict the properties of the material out of the atomic number, that is no easy question.

II - Then we will embark on a detailed metal genesis discussion, in the universe in general, and our planet Earth in particular, featuring stars, their origins, functioning and collapse, and then planets, and what happens there, chemistry and geology.

III - With the origins, distribution and main properties of the metals and their compounds understood, we will discuss then the basics of metallurgy, that is, extracting metals from their compounds, from the ancient times, and with a few modern aspects too.

IV - Finally, we will focus on iron and steel, with a lengthy theoretical discussion about both, with iron being the most important metal, physically speaking, and steel being the most important material, to mankind, and what can be done with them.

In the hope that you will find this book useful. Missing from our presentation will be modern technology like electricity passing through our metals, but is really electricity the key to a better world, that remains to be determined. In any case, there is agreement between the remaining wild tribes, the modern survivalists, and countless other communities that love nature, on a regular or even daily basis, that there is little interest in building an electric generator or chemical battery, out there in the wild. And we will take this principle, and cherish and adopt it, for the needs of the present book.

Many thanks go to the internet, sometimes when you fail with certain things, online videos are there for proving the contrary, and restoring the faith in yourself, and in mankind. Thanks as well to my cats, so many things to be learned from them, too.

Contents

Preface	3
Part I. Metals	9
Chapter 1. Physics, metals	11
1a. Basic physics	11
1b. Atomic theory	16
1c. Periodic table	22
1d. The metals	30
1e. Exercises	32
Chapter 2. Basic properties	33
2a. Solid state	33
2b. Magnetization	33
2c. Melting and beyond	33
2d. Heavy metals	33
2e. Exercises	33
Chapter 3. Stress, elasticity	35
3a. Elasticity	35
3b. Metal bars	35
3c. Sheet metal	35
3d. Springs	35
3e. Exercises	35
Chapter 4. Oxydation	37
4a. Molecules	37
4b. Oxygen	37
4c. Oxydation	37
4d. Basic compounds	37
4e. Exercises	37

Part II. Geology	39
Chapter 5. Stars, genesis	41
5a. High energy	41
5b. Atomic fusion	48
5c. Main sequence stars	48
5d. Star collapse	48
5e. Exercises	48
Chapter 6. Planet Earth	49
6a. Dust, planets	49
6b. Metallic core	49
6c. Basic geology	49
6d. Water effects	49
6e. Exercises	49
Chapter 7. Basic compounds	51
7a. Basic compounds	51
7b.	51
7c.	51
7d.	51
7e. Exercises	51
Chapter 8. Meteorites	53
8a. Meteorites	53
8b.	53
8c.	53
8d.	53
8e. Exercises	53
Part III. Metallurgy	55
Chapter 9. Stone Age	57
9a. Stone Age	57
9b.	57
9c.	57
9d.	57
9e. Exercises	57

CONTENTS

7

Chapter 10. Silver and gold	59
10a. Silver and gold	59
10b.	59
10c.	59
10d.	59
10e. Exercises	59
Chapter 11. Cooper, bronze	61
11a. Cooper, bronze	61
11b.	61
11c.	61
11d.	61
11e. Exercises	61
Chapter 12. Iron and steel	63
12a. Iron and steel	63
12b.	63
12c.	63
12d.	63
12e. Exercises	63
Part IV. Steel	65
Chapter 13. Iron, revised	67
13a. Iron, revised	67
13b.	67
13c.	67
13d.	67
13e. Exercises	67
Chapter 14. Steel, alloys	69
14a. Steel, alloys	69
14b.	69
14c.	69
14d.	69
14e. Exercises	69
Chapter 15. Carbon steel	71

15a. Carbon steel	71
15b.	71
15c.	71
15d.	71
15e. Exercises	71
Chapter 16. Axes and knives	73
16a. Axes and knives	73
16b.	73
16c.	73
16d.	73
16e. Exercises	73
Bibliography	75
Index	79

Part I

Metals

*These boots are made for walking
And that's just what they'll do
One of these days these boots
Are gonna walk all over you*

CHAPTER 1

Physics, metals

1a. Basic physics

Welcome to metals, and such a joy to have you here. We will learn many things in this book, some very ancient, some more modern, and with the whole thing having a unique flavor of physics, chemistry, geology, engineering, and history and social science too.

Starting with the basics, we would like to understand and identify the various metal ores, from the nature surrounding us, then get to extract the metal out of them, one way or another, and then use that metal for making all sorts of useful blades. Knives, axes, spades and other for working the land, why not swords too, and many more.

In practice now, before getting to this, metallurgy, there are many types of metals, and we first have to know more about them. Generally speaking, we would like our blades to be strong enough for doing the job, but with a certain amount of elasticity for not breaking, and also with an amount of softness for being easily sharpened, and also, if possible, with a certain resistance to corrosion. Does not look easy all this, but as a conclusion, speaking now more scientifically, we will be mainly interested in the mechanical properties of the metals, and their alloys, and with a look at their basic chemical properties too.

So, it should be with some basic physics and chemistry that we should start our study. Periodic table, that you surely know about from school, then go examine all the metals appearing there, mostly from a mechanical perspective, that would be our first job:

QUESTION 1.1. What are the possible available metals, from the periodic table, and what are their mechanical properties, in relation with the needs of metallurgy?

Easy job, you would say, so good time for me author to go now do some experiments, and come back when done, with the answer. However, I am a bit afraid that this might take a few millenia, or at least a few millenia is what all this took, to mankind, to figure out. So, with due excuses, I would rather prefer instead to keep lying in my sofa, and write this book, armed with some theoretical physics knowledge, by following:

PLAN 1.2. We will learn about atoms, periodic table and metals, and only after we will embark on a mechanical study, which will be mainly theoretical, to start with.

Getting started for good now, our first objective, atoms, are no easy business, and require some knowledge of basic physics. So, let us start with this. We first have:

THEOREM 1.3. *Electrodynamics is governed by the formulae*

$$\langle \nabla, E \rangle = \frac{\rho}{\varepsilon_0} \quad , \quad \langle \nabla, B \rangle = 0$$

$$\nabla \times E = -\dot{B} \quad , \quad \nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \dot{E}$$

called Maxwell equations.

PROOF. This is something fundamental, appearing as a tricky mixture of physics facts and mathematical results, the idea being as follows:

(1) To start with, electrodynamics is the science of moving electrical charges. And this is something quite complicated, because unlike in classical mechanics, where the Newton law is good for both the static and the dynamic setting, the Coulomb law, which is actually very similar to the Newton law, does the job when the charges are static, but no longer describes well the situation when the charges are moving.

(2) The problem comes from the fact that moving charges produce magnetism, and with this being visible when putting together two electric wires, which will attract or repel, depending on orientation. Thus, in contrast with classical mechanics, where static or dynamic problems are described by a unique field, the gravitational one, in electrodynamics we have two fields, namely the electric field E , and the magnetic field B .

(3) Fortunately, there is a full set of equations relating the electric field E and the magnetic field B , those above. Regarding the math, the dots denote derivatives with respect to time, and ∇ is the gradient operator, or space derivative, given by:

$$\nabla = \begin{pmatrix} \frac{d}{dx} \\ \frac{d}{dy} \\ \frac{d}{dz} \end{pmatrix}$$

(4) Regarding the physics, the first formula is the Gauss law, ρ being the charge, and ε_0 being a constant, and with this Gauss law more or less replacing the Coulomb law from electrostatics. The second formula is something basic, and anonymous. The third formula is the Faraday law. As for the fourth formula, this is the Ampère law, as modified by Maxwell, with J being the volume current density, and μ_0 being a constant. \square

Quite surprisingly, the constants μ_0, ε_0 appearing above are related as follows:

FACT 1.4. *The constants μ_0, ε_0 are related by the Biot-Savart formula*

$$\mu_0 \varepsilon_0 = \frac{1}{c^2}$$

with $c = 299,792,458$ being as usual the speed of light in vacuum.

The above formula suggests a subtle relation between electromagnetism and light, and indeed, we have here the following key theoretical result, to start with:

THEOREM 1.5. *In regions of space where there is no charge or current present the Maxwell equations for electrodynamics read*

$$\langle \nabla, E \rangle = \langle \nabla, B \rangle = 0$$

$$\nabla \times E = -\dot{B} \quad , \quad \nabla \times B = \dot{E}/c^2$$

and both the electric field E and magnetic field B are subject to the wave equation

$$\ddot{\varphi} = c^2 \Delta \varphi$$

where $\Delta = \sum_i d^2/dx_i^2$ is the Laplace operator, and c is the speed of light.

PROOF. Under the circumstances in the statement, namely no charge or current present, the Maxwell equations in Theorem 1.3 simply read:

$$\langle \nabla, E \rangle = \langle \nabla, B \rangle = 0$$

$$\nabla \times E = -\dot{B} \quad , \quad \nabla \times B = \dot{E}/c^2$$

Now by applying the curl operator to the last two equations, we obtain:

$$\nabla \times (\nabla \times E) = -\nabla \times \dot{B} = -(\nabla \times B)' = -\ddot{E}/c^2$$

$$\nabla \times (\nabla \times B) = \nabla \times \dot{E}/c^2 = (\nabla \times E)'/c^2 = -\ddot{B}/c^2$$

But the double curl operator is subject to the following formula:

$$\nabla \times (\nabla \times \varphi) = \nabla \langle \nabla, \varphi \rangle - \Delta \varphi$$

Now by using the first two equations, we are led to the conclusion in the statement. \square

So, what is light? Light is the wave predicted by Theorem 1.5, travelling in vacuum at the maximum possible speed, c , and with an important extra property being that it depends on a real positive parameter, that can be called, upon taste, frequency, wavelength, or color. And in what regards the creation of light, the mechanism here is as follows:

FACT 1.6. *An accelerating or decelerating charge produces electromagnetic radiation, called light, whose frequency and wavelength can be explicitly computed.*

This phenomenon can be observed in a variety of situations, such as the usual light bulbs, where electrons get decelerated by the filament, acting as a resistor, or in usual fire, which is a chemical reaction, with the electrons moving around, as they do in any chemical reaction, or in more complicated machinery like nuclear plants, particle accelerators, and so on, leading there to all sorts of eerie glows, of various colors.

Moving ahead, let us go back to the wave equation $\ddot{\varphi} = v^2 \Delta \varphi$ from Theorem 1.5, and try to understand its simplest solutions. In 1D, the situation is as follows:

THEOREM 1.7. *The 1D wave equation has as basic solutions the functions*

$$\varphi(x) = A \cos(kx - wt + \delta)$$

with A being called amplitude, $kx - wt + \delta$ being called the phase, k being the wave number, w being the angular frequency, and δ being the phase constant. We have

$$\lambda = \frac{2\pi}{k} \quad , \quad T = \frac{2\pi}{kv} \quad , \quad \nu = \frac{1}{T} \quad , \quad w = 2\pi\nu$$

relating the wavelength λ , period T , frequency ν , and angular frequency w . Moreover, any solution of the wave equation appears as a linear combination of such basic solutions.

PROOF. There are several things going on here, the idea being as follows:

(1) Our first claim is that the function φ in the statement satisfies indeed the wave equation, with speed $v = w/k$. For this purpose, observe that we have:

$$\ddot{\varphi} = -w^2\varphi \quad , \quad \frac{d^2\varphi}{dx^2} = -k^2\varphi$$

Thus, the wave equation is indeed satisfied, with speed $v = w/k$:

$$\ddot{\varphi} = \left(\frac{w}{k}\right)^2 \frac{d^2\varphi}{dx^2} = v^2 \frac{d^2\varphi}{dx^2}$$

(2) Regarding now the other things in the statement, all this is basically terminology, which is very natural, when thinking how $\varphi(x) = A \cos(kx - wt + \delta)$ propagates. As for the last assertion, this is something standard, coming from Fourier analysis. \square

As a first observation, the above result invites the use of complex numbers. Indeed, we can write the solutions that we found in a more convenient way, as follows:

$$\varphi(x) = \operatorname{Re} [A e^{i(kx - wt + \delta)}]$$

And we can in fact do even better, by absorbing the quantity $e^{i\delta}$ into the amplitude A , which becomes now a complex number, and writing our formula as:

$$\varphi = \operatorname{Re}(\tilde{\varphi}) \quad , \quad \tilde{\varphi} = \tilde{A} e^{i(kx - wt)}$$

Moving ahead now towards electromagnetism and 3D, let us formulate:

DEFINITION 1.8. *A monochromatic plane wave is a solution of the 3D wave equation which moves in only 1 direction, making it in practice a solution of the 1D wave equation, and which is of the special form found in Theorem 1.7, with no frequencies mixed.*

In other words, we are making here two assumptions on our wave. First is the 1-dimensionality assumption, which gets us into the framework of Theorem 1.7. And second is the assumption, in connection with the Fourier decomposition result from the end of Theorem 1.7, that our solution is of “pure” type, meaning a wave having a well-defined wavelength and frequency, instead of being a “packet” of such pure waves.

All this is still mathematics, and making now the connection with physics and electromagnetism, and more specifically with Theorem 1.5 and Fact 1.6, we have:

FACT 1.9. *Physically speaking, a monochromatic plane wave is the electromagnetic radiation appearing as in Theorem 1.5 and Fact 1.6, via equations of type*

$$E = \operatorname{Re}(\tilde{E}) \quad : \quad \tilde{E} = \tilde{E}_0 e^{i(\langle k, x \rangle - wt)}$$

$$B = \operatorname{Re}(\tilde{B}) \quad : \quad \tilde{B} = \tilde{B}_0 e^{i(\langle k, x \rangle - wt)}$$

with the wave number being now a vector, $k \in \mathbb{R}^3$. Moreover, it is possible to add to this an extra parameter, accounting for the possible polarization of the wave.

To be more precise, what we are doing here is to import the conclusions of our mathematical discussion so far, from Theorem 1.7 and Definition 1.8, into the context of our original physics discussion, from Fact 1.6. And also to add an extra twist coming from physics, and more specifically from the notion of polarization. More on this later.

In any case, we have now a decent intuition about what light is, and more on this later, and let us discuss now the examples. The idea is that we have various types of light, depending on frequency and wavelength. These are normally referred to as “electromagnetic waves”, but for keeping things simple and luminous, we will keep using the familiar term “light”. The classification, in a rough form, is as follows:

Frequency	Type	Wavelength
	—	
$10^{18} - 10^{20}$	γ rays	$10^{-12} - 10^{-10}$
$10^{16} - 10^{18}$	X-rays	$10^{-10} - 10^{-8}$
$10^{15} - 10^{16}$	UV	$10^{-8} - 10^{-7}$
	—	
$10^{14} - 10^{15}$	blue	$10^{-7} - 10^{-6}$
$10^{14} - 10^{15}$	yellow	$10^{-7} - 10^{-6}$
$10^{14} - 10^{15}$	red	$10^{-7} - 10^{-6}$
	—	
$10^{11} - 10^{14}$	IR	$10^{-6} - 10^{-3}$
$10^9 - 10^{11}$	microwave	$10^{-3} - 10^{-1}$
$1 - 10^9$	radio	$10^{-1} - 10^8$

Observe the tiny space occupied by the visible light, all colors there, and the many more missing, being squeezed under the $10^{14} - 10^{15}$ frequency banner. Here is a zoom on

that part, with of course the remark that all this, colors, is something subjective:

Frequency THz = 10^{12} Hz	Color	Wavelength nm = 10^{-9} m
	—	
670 – 790	violet	380 – 450
620 – 670	blue	450 – 485
600 – 620	cyan	485 – 500
530 – 600	green	500 – 565
510 – 530	yellow	565 – 590
480 – 510	orange	590 – 625
400 – 480	red	625 – 750

Outside visible light we have, as you probably know it, UV on higher frequencies, and IR on lower frequencies. At the high frequency end we have X-rays, that you surely know about too, and γ rays, which are usually associated with various bad things, such as thunderstorms, solar flares, and small bugs with our nuclear energy technology.

As for the lower frequency end of the scale, first we have microwaves, but if you love physics and chemistry you should learn some cooking, that's first-class chemistry, that you can practice every day. And then we have all sorts of radio wavelenghts, including FM, followed by AM, and then by several more obscure low-frequency waves.

Importantly, both ends of the table are a bit loose. At the high frequency end there are some restrictions coming from quantum mechanics, and more on them later. As for the low frequency end, what's wave and what's not is a bit of a philosophical question.

1b. Atomic theory

Back now towards our business, atomic theory, with the above in hand, we can do some useful optics. Light usually comes in “bundles”, with waves of several wavelenghts coming at the same time, from the same source, and the first challenge is that of separating these wavelenghts. In order to discuss this, let us start with the following fact:

FACT 1.10. *Inside a linear, homogeneous medium, where there is no free charge or current present, the Maxwell equations for electrodynamics read*

$$\begin{aligned} \langle \nabla, E \rangle &= \langle \nabla, B \rangle = 0 \\ \nabla \times E &= -\dot{B} \quad , \quad \nabla \times B = \varepsilon \mu \dot{E} \end{aligned}$$

with E, B being as before the electric and the magnetic field, and with $\varepsilon > \varepsilon_0$ and $\mu > \mu_0$ being the electric permittivity and magnetic permeability of the medium.

Observe that this statement is precisely the first part of Theorem 1.5, with the vacuum constants ε_0, μ_0 being now replaced by their versions ε, μ , concerning the medium in question. In what regards now the second part of Theorem 1.5, we have:

THEOREM 1.11. *Inside a linear, homogeneous medium, where there is no free charge or free current present, both E and B are subject to the wave equation*

$$\ddot{\varphi} = v^2 \Delta \varphi$$

with v being the speed of light inside the medium, given by

$$v = \frac{c}{n} \quad : \quad n = \sqrt{\frac{\epsilon\mu}{\epsilon_0\mu_0}}$$

with the quantity on the right $n > 1$ being called *refraction index of the medium*.

PROOF. This is something that we know well in vacuum, from the above, and the proof in general is identical, with the resulting speed being:

$$v = \frac{1}{\sqrt{\epsilon\mu}}$$

But this formula can be written in a more familiar form, as above. □

Next in line, and of interest for us, as we will soon discover, we have:

FACT 1.12. *When travelling through a material, and hitting a new material, some of the light gets reflected, at the same angle, and some of it gets refracted, at a different angle, depending both on the old and the new material, and on the wavelength.*

Again, this is something deep, and there are many things that can be said here. As a basic formula, we have the famous Snell law, which relates the incidence angle θ_1 to the refraction angle θ_2 , via the following simple formula:

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{n_1(\lambda)}{n_2(\lambda)}$$

Here $n_i(\lambda)$ are the refraction indices of the two materials, adjusted for the wavelength, and with this adjustment for wavelength being the whole point, which is something quite complicated. For an introduction to all this, we refer for instance to Griffiths [40].

As a simple consequence of the above, of great practical interest, we have:

THEOREM 1.13. *Light can be decomposed, by using a prism.*

PROOF. This follows from Fact 1.12. Indeed, when hitting a piece of glass, provided that the hitting angle is not 90° , the light will decompose over the wavelengths present, with the corresponding refraction angles depending on these wavelengths. And we can capture these split components at the exit from the piece of glass, again deviated a bit, provided that the exit surface is not parallel to the entry surface. And the simplest device doing the job, that is, having two non-parallel faces, is a prism. □

With this in hand, we can now talk about spectroscopy:

FACT 1.14. *We can study events via spectroscopy, by capturing the light the event has produced, decomposing it with a prism, carefully recording its “spectral signature”, consisting of the wavelengths present, and their density, and then doing some reverse engineering, consisting in reconstructing the event out of its spectral signature.*

Good news, we can now discuss atomic theory. There is a long story here, involving many discoveries, around 1890-1900, focusing on hydrogen H. We will present here things a bit retrospectively. First on our list is the following discovery, by Lyman in 1906:

FACT 1.15 (Lyman). *The hydrogen atom has spectral lines given by the formula*

$$\frac{1}{\lambda} = R \left(1 - \frac{1}{n^2} \right)$$

where $R \simeq 1.097 \times 10^7$ and $n \geq 2$, which are as follows,

n	Name	Wavelength	Color
—	—	—	—
2	α	121.567	UV
3	β	102.572	UV
4	γ	97.254	UV
\vdots	\vdots	\vdots	\vdots
∞	limit	91.175	UV

called Lyman series of the hydrogen atom.

Observe that all the Lyman series lies in UV, which is invisible to the naked eye. The first discovery, which was the big one, and the breakthrough, was by Balmer, the founding father of all this, back in 1885, in the visible range, as follows:

FACT 1.16 (Balmer). *The hydrogen atom has spectral lines given by the formula*

$$\frac{1}{\lambda} = R \left(\frac{1}{4} - \frac{1}{n^2} \right)$$

where $R \simeq 1.097 \times 10^7$ and $n \geq 3$, which are as follows,

n	Name	Wavelength	Color
—	—	—	—
3	α	656.279	red
4	β	486.135	aqua
5	γ	434.047	blue
6	δ	410.173	violet
7	ε	397.007	UV
\vdots	\vdots	\vdots	\vdots
∞	limit	346.600	UV

called Balmer series of the hydrogen atom.

So, this was Balmer's original result, which started everything. As a third main result now, this time in IR, due to Paschen in 1908, we have:

FACT 1.17 (Paschen). *The hydrogen atom has spectral lines given by the formula*

$$\frac{1}{\lambda} = R \left(\frac{1}{9} - \frac{1}{n^2} \right)$$

where $R \simeq 1.097 \times 10^7$ and $n \geq 4$, which are as follows,

n	Name	Wavelength	Color
	—	—	
4	α	1875	IR
5	β	1282	IR
6	γ	1094	IR
\vdots	\vdots	\vdots	\vdots
∞	limit	820.4	IR

called Paschen series of the hydrogen atom.

Observe the striking similarity between the above three results. In fact, we have here the following fundamental, grand result, due to Rydberg in 1888, based on the Balmer series, and with later contributions by Ritz in 1908, using the Lyman series as well:

CONCLUSION 1.18 (Rydberg, Ritz). *The spectral lines of the hydrogen atom are given by the Rydberg formula, depending on integer parameters $n_1 < n_2$,*

$$\frac{1}{\lambda_{n_1 n_2}} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

with R being the Rydberg constant for hydrogen, which is as follows:

$$R \simeq 1.096\,775\,83 \times 10^7$$

These spectral lines combine according to the Ritz-Rydberg principle, as follows:

$$\frac{1}{\lambda_{n_1 n_2}} + \frac{1}{\lambda_{n_2 n_3}} = \frac{1}{\lambda_{n_1 n_3}}$$

Similar formulae hold for other atoms, with suitable fine-tunings of R .

Here the first part, the Rydberg formula, generalizes the results of Lyman, Balmer, Paschen, which appear at $n_1 = 1, 2, 3$, at least retrospectively. The Rydberg formula predicts further spectral lines, appearing at $n_1 = 4, 5, 6, \dots$, and these were discovered later, by Brackett in 1922, Pfund in 1924, Humphreys in 1953, and others afterwards,

with all these extra lines being in far IR. The simplified complete table is as follows:

n_1	n_2	Series name	Wavelength $n_2 = \infty$	Color $n_2 = \infty$
1	2 – ∞	Lyman	91.13 nm	UV
2	3 – ∞	Balmer	364.51 nm	UV
3	4 – ∞	Paschen	820.14 nm	IR
4	5 – ∞	Brackett	1458.03 nm	far IR
5	6 – ∞	Pfund	2278.17 nm	far IR
6	7 – ∞	Humphreys	3280.56 nm	far IR
\vdots	\vdots	\vdots	\vdots	\vdots

Regarding the last assertion, concerning other elements, this is something conjectured and partly verified by Ritz, and fully verified and clarified later, via many experiments, the fine-tuning of R being basically $R \rightarrow RZ^2$, where Z is the atomic number.

From a theoretical physics viewpoint, the main result remains the middle assertion, called Ritz-Rydberg combination principle, which is something quite puzzling. But this combination principle reminds the formula $e_{n_1 n_2} e_{n_2 n_3} = e_{n_1 n_3}$ for the usual matrix units $e_{ij} : e_j \rightarrow e_i$. Thus, we are in familiar territory here, and we can start dreaming of:

THOUGHT 1.19. *Observables in quantum mechanics should be some sort of infinite matrices, generalizing the Lyman, Balmer, Paschen lines of the hydrogen atom, and multiplying between them as the matrices do, as to produce further observables.*

Now back to more concrete things, in relation with this, as a main problem that we would like to solve, we have the understanding the intimate structure of matter, at the atomic level. There is of course a long story here, regarding the intimate structure of matter, going back centuries and even millennia ago, and our presentation here will be quite simplified. As a starting point, let us agree on the following claim:

CLAIM 1.20. *Ordinary matter is made of small particles called atoms, with each atom appearing as a mix of even smaller particles, namely protons +, neutrons 0 and electrons –, with the same number of protons + and electrons –.*

As a first observation, this is something which does not look obvious at all, with probably lots of work, by many people, being involved, as to lead to this claim. And so it is. The story goes back to the discovery of charges and electricity, which were attributed to a small particle, the electron –. Now since matter is by default neutral, this naturally leads to the consideration to the proton +, having the same charge as the electron.

But, as a natural question, why should be these electrons – and protons + that small? And also, what about the neutron 0? These are not easy questions, and the fact that it

is so came from several clever experiments. Let us first recall that careful experiments with tiny particles are practically impossible. However, all sorts of brutal experiments, such as bombarding matter with other pieces of matter, accelerated to the extremes, or submitting it to huge electric and magnetic fields, do work. And it is such kind of experiments, due to Thomson, Rutherford and others, “peeling off” protons $+$, neutrons 0 and electrons $-$ from matter, and observing them, that led to the conclusion that these small beasts $+$, 0 , $-$ exist indeed, in agreement with Claim 1.20.

Of particular importance here was as well the radioactivity theory of Becquerel and Pierre and Marie Curie, involving this time such small beasts, or perhaps some related radiation, peeling off by themselves, in heavy elements such as uranium ${}_{92}\text{U}$, polonium ${}_{84}\text{Po}$ and radium ${}_{88}\text{Ra}$. And there was also Einstein’s work on the photoelectric effect, light interacting with matter, suggesting that even light itself might have associated to it some kind of particle, called photon. All this goes of course beyond Claim 1.20, with further particles involved, and more on this later, but as a general idea, all this deluge of small particle findings, all coming around 1900-1910, further solidified Claim 1.20.

So, taking now Claim 1.20 for granted, how are then the atoms organized, as mixtures of protons $+$, neutrons 0 and electrons $-$? The answer here lies again in the above-mentioned “brutal” experiments of Thomson, Rutherford and others, which not only proved Claim 1.20, but led to an improved version of it, as follows:

CLAIM 1.21. *The atoms are formed by a core of protons $+$ and neutrons 0 , surrounded by a cloud of electrons $-$, gravitating around the core.*

This is a considerable advance, because we are now into familiar territory, namely some kind of mechanics. And with this in mind, all the pieces of our puzzle start fitting together, and we are led to the following grand conclusion:

CLAIM 1.22 (Bohr and others). *The atoms are formed by a core of protons and neutrons, surrounded by a cloud of electrons, basically obeying to a modified version of electromagnetism. And with a fine mechanism involved, as follows:*

- (1) *The electrons are free to move only on certain specified elliptic orbits, labelled $1, 2, 3, \dots$, situated at certain specific heights.*
- (2) *The electrons can jump or fall between orbits $n_1 < n_2$, absorbing or emitting light and heat, that is, electromagnetic waves, as accelerating charges.*
- (3) *The energy of such a wave, coming from $n_1 \rightarrow n_2$ or $n_2 \rightarrow n_1$, is given, via the Planck viewpoint, by the Rydberg formula, applied with $n_1 < n_2$.*
- (4) *The simplest such jumps are those observed by Lyman, Balmer, Paschen. And multiple jumps explain the Ritz-Rydberg formula.*

And isn’t this beautiful. Moreover, some further claims, also by Bohr and others, are that the theory can be further extended and fine-tuned as to explain many other

phenomena, such as the above-mentioned findings of Einstein, and of Becquerel and Pierre and Marie Curie, and generally speaking, all the physics and chemistry known.

And the story is not over here. Following now Heisenberg, the next claim is that the underlying mathematics in all the above can lead to a beautiful axiomatization of quantum mechanics, as a “matrix mechanics”, along the lines of Thought 1.19.

1c. Periodic table

Before getting into Heisenberg matrix mechanics, let us hear as well the point of view of Schrödinger, which came a few years later. His idea was to forget about exact things, and try to investigate the hydrogen atom statistically. Let us start with:

QUESTION 1.23. *In the context of the hydrogen atom, assuming that the proton is fixed, what is the probability density $\varphi_t(x)$ of the position of the electron e , at time t ,*

$$P_t(e \in V) = \int_V \varphi_t(x) dx$$

as function of an initial probability density $\varphi_0(x)$? Moreover, can the corresponding equation be solved, and will this prove the Bohr claims for hydrogen, statistically?

In order to get familiar with this question, let us first look at examples coming from classical mechanics. In the context of a particle whose position at time t is given by $x_0 + \gamma(t)$, the evolution of the probability density will be given by:

$$\varphi_t(x) = \varphi_0(x + \gamma(t))$$

However, such examples are somewhat trivial, of course not in relation with the computation of γ , usually a difficult question, but in relation with our questions, and do not apply to the electron. The point indeed is that, in what regards the electron, we have:

FACT 1.24. *In respect with various simple interference experiments:*

- (1) *The electron is definitely not a particle in the usual sense.*
- (2) *But in most situations it behaves exactly like a wave.*
- (3) *But in other situations it behaves like a particle.*

Getting back now to the Schrödinger question, all this suggests to use, as for the waves, an amplitude function $\psi_t(x) \in \mathbb{C}$, related to the density $\varphi_t(x) > 0$ by the formula $\varphi_t(x) = |\psi_t(x)|^2$. Not that a big deal, you would say, because the two are related by simple formulae as follows, with $\theta_t(x)$ being an arbitrary phase function:

$$\varphi_t(x) = |\psi_t(x)|^2 \quad , \quad \psi_t(x) = e^{i\theta_t(x)} \sqrt{\varphi_t(x)}$$

However, such manipulations can be crucial, raising for instance the possibility that the amplitude function satisfies some simple equation, while the density itself, maybe not. And this is what happens indeed. Schrödinger was led in this way to:

CLAIM 1.25 (Schrödinger). *In the context of the hydrogen atom, the amplitude function of the electron $\psi = \psi_t(x)$ is subject to the Schrödinger equation*

$$i\hbar\dot{\psi} = -\frac{\hbar^2}{2m}\Delta\psi + V\psi$$

m being the mass, $\hbar = h_0/2\pi$ the reduced Planck constant, and V the Coulomb potential of the proton. The same holds for movements of the electron under any potential V .

Observe the similarity with the wave equation $\ddot{\varphi} = v^2\Delta\varphi$, and with the heat equation $\dot{\varphi} = \alpha\Delta\varphi$ too. Many things can be said here. Following now Heisenberg and Schrödinger, and then especially Dirac, who did the axiomatization work, we have:

DEFINITION 1.26. *In quantum mechanics the states of the system are vectors of a Hilbert space H , and the observables of the system are linear operators*

$$T : H \rightarrow H$$

which can be densely defined, and are taken self-adjoint, $T = T^$. The average value of such an observable T , evaluated on a state $\xi \in H$, is given by:*

$$\langle T \rangle = \langle T\xi, \xi \rangle$$

In the context of the Schrödinger mechanics of the hydrogen atom, the Hilbert space is the space $H = L^2(\mathbb{R}^3)$ where the wave function ψ lives, and we have

$$\langle T \rangle = \int_{\mathbb{R}^3} T(\psi) \cdot \bar{\psi} dx$$

which is called “sandwiching” formula, with the operators

$$x \quad , \quad -\frac{i\hbar}{m}\nabla \quad , \quad -i\hbar\nabla \quad , \quad -\frac{\hbar^2\Delta}{2m} \quad , \quad -\frac{\hbar^2\Delta}{2m} + V$$

representing the position, speed, momentum, kinetic energy, and total energy.

In other words, we are doing here two things. First, we are declaring by axiom that various “sandwiching” formulae found before by Heisenberg, involving the operators at the end, that we will not get into in detail here, hold true. And second, we are raising the possibility for other quantum mechanical systems, more complicated, to be described as well by the mathematics of the operators on a certain Hilbert space H , as above.

Moving ahead now towards hydrogen, we are interested in the case where V is the usual quadratic Coulomb potential of the proton, given by the following formula:

$$V = -\frac{e^2}{4\pi\epsilon_0} \cdot \frac{1}{r}$$

This potential is time-independent, and we have the following result:

THEOREM 1.27. *In the case of time-independent potentials V , including the Coulomb potential of the proton, the solutions of the Schrödinger equation*

$$i\hbar\dot{\psi} = -\frac{\hbar^2}{2m}\Delta\psi + V\psi$$

which are of the following special form, with the time and space variables separated,

$$\psi_t(x) = w_t\phi(x)$$

are given by the following formulae, with E being a certain constant,

$$w = e^{-iEt/\hbar}w_0 \quad , \quad E\phi = -\frac{\hbar^2}{2m}\Delta\phi + V\phi$$

with the equation for ϕ being called time-independent Schrödinger equation.

PROOF. By dividing by ψ , the Schrödinger equation becomes:

$$i\hbar \cdot \frac{\dot{w}}{w} = -\frac{\hbar^2}{2m} \cdot \frac{\Delta\phi}{\phi} + V$$

Now since the left-hand side depends only on time, and the right-hand side depends only on space, both quantities must equal a constant E , and this gives the result. \square

Moving ahead with theory, we can further build on Theorem 1.27, with a number of key observations on the time-independent Schrödinger equation, as follows:

THEOREM 1.28. *In the case of time-independent potentials V , the Schrödinger equation and its time-independent version have the following properties:*

- (1) *For solutions of type $\psi = w_t\phi(x)$, the density $\varphi = |\psi|^2$ is time-independent, and more generally, all quantities of type $\langle T \rangle$ are time-independent.*
- (2) *The time-independent Schrödinger equation can be written as $\widehat{H}\phi = E\phi$, with $H = T + V$ being the total energy, of Hamiltonian.*
- (3) *For solutions of type $\psi = w_t\phi(x)$ we have $\langle H^k \rangle = E^k$ for any k . In particular we have $\langle H \rangle = E$, and the variance is $\langle H^2 \rangle - \langle H \rangle^2 = 0$.*

PROOF. All the formulae are clear indeed from the fact that, when using the sandwiching formula for computing averages, the phases will cancel:

$$\langle T \rangle = \int_{\mathbb{R}^3} \bar{\psi} \cdot T \cdot \psi \, dx = \int_{\mathbb{R}^3} \bar{\phi} \cdot T \cdot \phi \, dx$$

Thus, we are led to the various conclusions in the statement. \square

Regarding now hydrogen, the story here is quite long, involving a lot of mathematics, and we have the following result, which proves the original claim by Bohr:

THEOREM 1.29 (Schrödinger). *In the case of the hydrogen atom, where V is the Coulomb potential of the proton, the modified radial equation, which reads*

$$Eu = -\frac{\hbar^2}{2m} \cdot u'' + \left(-\frac{Ke^2}{r} + \frac{\hbar^2 l(l+1)}{2mr^2} \right) u$$

leads to the Bohr formula for allowed energies,

$$E_n = -\frac{m}{2} \left(\frac{Ke^2}{\hbar} \right)^2 \cdot \frac{1}{n^2}$$

with $n \in \mathbb{N}$, the binding energy being

$$E_1 \simeq -2.177 \times 10^{-18}$$

with means $E_1 \simeq -13.591$ eV.

PROOF. This is again something non-trivial, and we will be following Griffiths [41], with some details missing. The idea is as follows:

(1) By dividing our modified radial equation by E , this becomes:

$$-\frac{\hbar^2}{2mE} \cdot u'' = \left(1 + \frac{Ke^2}{Er} - \frac{\hbar^2 l(l+1)}{2mEr^2} \right) u$$

In terms of $\alpha = \sqrt{-2mE}/\hbar$, this equation takes the following form:

$$\frac{u''}{\alpha^2} = \left(1 + \frac{Ke^2}{Er} + \frac{l(l+1)}{(\alpha r)^2} \right) u$$

In terms of the new variable $p = \alpha r$, this latter equation reads:

$$u'' = \left(1 + \frac{\alpha Ke^2}{Ep} + \frac{l(l+1)}{p^2} \right) u$$

Now let us introduce a new constant S for our problem, as follows:

$$S = -\frac{\alpha Ke^2}{E}$$

In terms of this new constant, our equation reads:

$$u'' = \left(1 - \frac{S}{p} + \frac{l(l+1)}{p^2} \right) u$$

(2) The idea will be that of looking for a solution written as a power series, but before that, we must “peel off” the asymptotic behavior. Which is something that can be done, of course, heuristically. With $p \rightarrow \infty$ we are led to $u'' = u$, and ignoring the solution $u = e^p$ which blows up, our approximate asymptotic solution is:

$$u \sim e^{-p}$$

Similarly, with $p \rightarrow 0$ we are led to $u'' = l(l+1)u/p^2$, and ignoring the solution $u = p^{-l}$ which blows up, our approximate asymptotic solution is:

$$u \sim p^{l+1}$$

(3) The above heuristic considerations suggest writing our function u as follows:

$$u = p^{l+1}e^{-p}v$$

So, let us do this. In terms of v , we have the following formula:

$$u' = p^l e^{-p} [(l+1-p)v + pv']$$

Differentiating a second time gives the following formula:

$$u'' = p^l e^{-p} \left[\left(\frac{l(l+1)}{p} - 2l - 2 + p \right) v + 2(l+1-p)v' + pv'' \right]$$

Thus the radial equation, as modified in (1) above, reads:

$$pv'' + 2(l+1-p)v' + (S - 2(l+1))v = 0$$

(4) We will be looking for a solution v appearing as a power series:

$$v = \sum_{j=0}^{\infty} c_j p^j$$

But our equation leads to the following recurrence formula for the coefficients:

$$c_{j+1} = \frac{2(j+l+1) - S}{(j+1)(j+2l+2)} \cdot c_j$$

(5) We are in principle done, but we still must check that, with this choice for the coefficients c_j , our solution v , or rather our solution u , does not blow up. And the whole point is here. Indeed, at $j \gg 0$ our recurrence formula reads, approximately:

$$c_{j+1} \simeq \frac{2c_j}{j}$$

But, surprisingly, this leads to $v \simeq c_0 e^{2p}$, and so to $u \simeq c_0 p^{l+1} e^p$, which blows up.

(6) As a conclusion, the only possibility for u not to blow up is that where the series defining v terminates at some point. Thus, we must have for a certain index j :

$$2(j+l+1) = S$$

In other words, we must have, for a certain integer $n > l$:

$$S = 2n$$

(7) We are almost there. Recall from (1) above that S was defined as follows:

$$S = -\frac{\alpha K e^2}{E} \quad : \quad \alpha = \frac{\sqrt{-2mE}}{h}$$

Thus, we have the following formula for the square of S :

$$S^2 = \frac{\alpha^2 K^2 e^4}{E^2} = -\frac{2mE}{h^2} \cdot \frac{K^2 e^4}{E^2} = -\frac{2mK^2 e^4}{h^2 E}$$

Now by using the formula $S = 2n$ from (6), the energy E must be of the form:

$$E = -\frac{2mK^2 e^4}{h^2 S^2} = -\frac{mK^2 e^4}{2h^2 n^2}$$

Calling this energy E_n , depending on $n \in \mathbb{N}$, we have, as claimed:

$$E_n = -\frac{m}{2} \left(\frac{Ke^2}{h} \right)^2 \cdot \frac{1}{n^2}$$

(8) Thus, we proved the Bohr formula. Regarding numerics, the data is as follows:

$$\begin{aligned} K &= 8.988 \times 10^9 \quad , \quad e = 1.602 \times 10^{-19} \\ h &= 1.055 \times 10^{-34} \quad , \quad m = 9.109 \times 10^{-31} \end{aligned}$$

But this gives the formula of E_1 in the statement. □

As a first remark, all this agrees with the Rydberg formula, due to:

THEOREM 1.30. *The Rydberg constant for hydrogen is given by*

$$R = -\frac{E_1}{h_0 c}$$

where E_1 is the Bohr binding energy, and the Rydberg formula itself, namely

$$\frac{1}{\lambda_{n_1 n_2}} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

simply reads, via the energy formula in Theorem 1.29,

$$\frac{1}{\lambda_{n_1 n_2}} = \frac{E_{n_2} - E_{n_1}}{h_0 c}$$

which is in agreement with the Planck formula $E = h_0 c / \lambda$.

PROOF. Here the first assertion is something numeric, coming from the fact that the formula in the statement gives, when evaluated, the Rydberg constant:

$$R = \frac{-E_1}{h_0 c} = \frac{2.177 \times 10^{-18}}{6.626 \times 10^{-34} \times 2.998 \times 10^8} = 1.096 \times 10^7$$

Regarding now the second assertion, by dividing $R = -E_1/(h_0 c)$ by any number of type n^2 we obtain, according to the energy convention in Theorem 1.29:

$$\frac{R}{n^2} = -\frac{E_n}{h_0 c}$$

But these are exactly the numbers which are subject to subtraction in the Rydberg formula, and so we are led to the conclusion in the statement. □

Let us investigate now the case of arbitrary atoms. We will need some general theory for the many-particle systems in quantum mechanics. Let us start with:

DEFINITION 1.31. *The wave function of a system of electrons e_1, \dots, e_Z , given by*

$$P_t(e_1 \in V_1, \dots, e_Z \in V_Z) = \int_{V_1 \times \dots \times V_Z} |\psi_t(x_1, \dots, x_Z)|^2 dx$$

is governed by the Schrödinger equation $i\hbar\dot{\psi} = \hat{H}\psi$, with Hamiltonian as follows,

$$\hat{H} = -\frac{\hbar}{2m} \sum_i \Delta_i + Ke^2 \sum_{i < j} \frac{1}{\|x_i - x_j\|} + V(x_1, \dots, x_Z)$$

with the middle sum standing for the Coulomb repulsions between them.

We are interested here in the case of atoms, where V is the Coulomb attraction potential coming from a Ze charge. Here the problem to be solved is as follows:

PROBLEM 1.32. *Consider an atom of atomic number Z , meaning a fixed Ze charge, surrounded by electrons e_1, \dots, e_Z . The problem is to solve the Schrödinger equation*

$$i\hbar\dot{\psi} = \hat{H}\psi$$

with Hamiltonian as follows,

$$\hat{H} = \sum_i \left(-\frac{\hbar}{2m} \Delta_i - \frac{KZe^2}{\|x_i\|} \right) + Ke^2 \sum_{i < j} \frac{1}{\|x_i - x_j\|}$$

or at least to understand how e_1, \dots, e_Z manage to live together, in a stable way.

A first idea would be of course that of ignoring the right term, Coulomb repulsion. In the simplest case, that of the helium atom, the situation is as follows:

FACT 1.33. *For the helium atom, $Z = 2$, ignoring the Coulomb repulsion between electrons leads, via separation of variables, to product wave functions*

$$\phi(x_1, x_2) = \phi'_{n_1 l_1 m_1}(x_1) \phi'_{n_2 l_2 m_2}(x_2)$$

with the prime signs standing for the doubling $e \rightarrow 2e$ of the central charge, with energies:

$$E_{n_1 n_2} = 4(E_{n_1} + E_{n_2})$$

This model predicts a ground state energy for helium given by

$$E_0 = 8 \times (-13.6) = -109 \text{ eV}$$

which is considerably smaller than the observed $E_0 = -79 \text{ eV}$.

Moving ahead, let us focus on a more modest question, that at the end of Problem 1.32, namely understanding how the electrons e_1, \dots, e_Z manage to live together. Here our method of ignoring the Coulomb repulsion between electrons is not that bad, and for helium for instance, we are led in this way to some interesting conclusions. For instance the excited states of helium must appear as products as follows:

$$\phi_{100}(x_1)\phi_{nlm}(x_2) \quad , \quad \phi_{nlm}(x_1)\phi_{100}(x_2)$$

And this is not very far from reality, and is actually quite close to reality, if we add spin to our discussion. Speaking now spin, this is indeed something that we ignored so far in the above. And spin is in fact a key component to our problem, because we have:

FACT 1.34 (Pauli exclusion principle). *Two electrons cannot occupy the same quantum numbers n, l, m , with same spin $s = \pm 1/2$.*

Good news, we have now all the ingredients for discussing the known atoms, or chemical elements, $Z = 1, \dots, 118$. These can be arranged in a table, as follows:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
1	$\frac{\text{H}}{1}$																	$\frac{\text{He}}{2}$	
2	$\frac{\text{Li}}{3}$	$\frac{\text{Be}}{4}$											$\frac{\text{B}}{5}$	$\frac{\text{C}}{6}$	$\frac{\text{N}}{7}$	$\frac{\text{O}}{8}$	$\frac{\text{F}}{9}$	$\frac{\text{Ne}}{10}$	
3	$\frac{\text{Na}}{11}$	$\frac{\text{Mg}}{12}$											$\frac{\text{Al}}{13}$	$\frac{\text{Si}}{14}$	$\frac{\text{P}}{15}$	$\frac{\text{S}}{16}$	$\frac{\text{Cl}}{17}$	$\frac{\text{Ar}}{18}$	
4	$\frac{\text{K}}{19}$	$\frac{\text{Ca}}{20}$	$\frac{\text{Sc}}{21}$	$\frac{\text{Ti}}{22}$	$\frac{\text{V}}{23}$	$\frac{\text{Cr}}{24}$	$\frac{\text{Mn}}{25}$	$\frac{\text{Fe}}{26}$	$\frac{\text{Co}}{27}$	$\frac{\text{Ni}}{28}$	$\frac{\text{Cu}}{29}$	$\frac{\text{Zn}}{30}$	$\frac{\text{Ga}}{31}$	$\frac{\text{Ge}}{32}$	$\frac{\text{As}}{33}$	$\frac{\text{Se}}{34}$	$\frac{\text{Br}}{35}$	$\frac{\text{Kr}}{36}$	
5	$\frac{\text{Rb}}{37}$	$\frac{\text{Sr}}{38}$	$\frac{\text{Y}}{39}$	$\frac{\text{Zr}}{40}$	$\frac{\text{Nb}}{41}$	$\frac{\text{Mo}}{42}$	$\frac{\text{Tc}}{43}$	$\frac{\text{Ru}}{44}$	$\frac{\text{Rh}}{45}$	$\frac{\text{Pd}}{46}$	$\frac{\text{Ag}}{47}$	$\frac{\text{Cd}}{48}$	$\frac{\text{In}}{49}$	$\frac{\text{Sn}}{50}$	$\frac{\text{Sb}}{51}$	$\frac{\text{Te}}{52}$	$\frac{\text{I}}{53}$	$\frac{\text{Xe}}{54}$	
6	$\frac{\text{Cs}}{55}$	$\frac{\text{Ba}}{56}$	<i>l</i>	$\frac{\text{Lu}}{71}$	$\frac{\text{Hf}}{72}$	$\frac{\text{Ta}}{73}$	$\frac{\text{W}}{74}$	$\frac{\text{Re}}{75}$	$\frac{\text{Os}}{76}$	$\frac{\text{Ir}}{77}$	$\frac{\text{Pt}}{78}$	$\frac{\text{Au}}{79}$	$\frac{\text{Hg}}{80}$	$\frac{\text{Tl}}{81}$	$\frac{\text{Pb}}{82}$	$\frac{\text{Bi}}{83}$	$\frac{\text{Po}}{84}$	$\frac{\text{At}}{85}$	$\frac{\text{Rn}}{86}$
7	$\frac{\text{Fr}}{87}$	$\frac{\text{Ra}}{88}$	<i>a</i>	$\frac{\text{Lr}}{103}$	$\frac{\text{Rf}}{104}$	$\frac{\text{Db}}{105}$	$\frac{\text{Sg}}{106}$	$\frac{\text{Bh}}{107}$	$\frac{\text{Hs}}{108}$	$\frac{\text{Mt}}{109}$	$\frac{\text{Ds}}{110}$	$\frac{\text{Rg}}{111}$	$\frac{\text{Cn}}{112}$	$\frac{\text{Nh}}{113}$	$\frac{\text{Fl}}{114}$	$\frac{\text{Mc}}{115}$	$\frac{\text{Lv}}{116}$	$\frac{\text{Ts}}{117}$	$\frac{\text{Og}}{118}$
			<i>l</i> :	$\frac{\text{La}}{57}$	$\frac{\text{Ce}}{58}$	$\frac{\text{Pr}}{59}$	$\frac{\text{Nd}}{60}$	$\frac{\text{Pm}}{61}$	$\frac{\text{Sm}}{62}$	$\frac{\text{Eu}}{63}$	$\frac{\text{Gd}}{64}$	$\frac{\text{Tb}}{65}$	$\frac{\text{Dy}}{66}$	$\frac{\text{Ho}}{67}$	$\frac{\text{Er}}{68}$	$\frac{\text{Tm}}{69}$	$\frac{\text{Yb}}{70}$		
			<i>a</i> :	$\frac{\text{Ac}}{89}$	$\frac{\text{Th}}{90}$	$\frac{\text{Pa}}{91}$	$\frac{\text{U}}{92}$	$\frac{\text{Np}}{93}$	$\frac{\text{Pu}}{94}$	$\frac{\text{Am}}{95}$	$\frac{\text{Cm}}{96}$	$\frac{\text{Bk}}{97}$	$\frac{\text{Cf}}{98}$	$\frac{\text{Es}}{99}$	$\frac{\text{Fm}}{100}$	$\frac{\text{Md}}{101}$	$\frac{\text{No}}{102}$		

Here the horizontal parameter $1, \dots, 18$ is called the group, and the vertical parameter $1, \dots, 7$ is called the period. The two rows on the bottom consist of lanthanum $_{57}\text{La}$ and its followers, called lanthanides, and of actinium $_{89}\text{Ac}$ and its followers, called actinides.

These are to be inserted in the main table, where indicated, lanthanides between barium $_{56}\text{Ba}$ and lutetium $_{71}\text{Lu}$, and actinides between radium $_{88}\text{Ra}$ and lawrencium $_{103}\text{Lr}$.

Thus, the periodic table, when correctly drawn, but no one does that because of obvious typographical reasons, is in fact a 7×32 table. Note here that, according to our 7×18 convention, which is the standard one, lanthanides and actinides don't have a group number $1, \dots, 18$. Their group is by definition "lanthanides" and "actinides".

1d. The metals

Let us explore now the periodic table, in look for metals. We will do this slowly. To start with, here are the elements up to krypton $_{36}\text{Kr}$, absolutely needed for everything:

DEFINITION 1.35. *The elements up to krypton $_{36}\text{Kr}$ are as follows:*

- (1) *Hydrogen $_1\text{H}$, helium $_2\text{He}$.*
- (2) *Lithium $_3\text{Li}$, beryllium $_4\text{Be}$, boron $_5\text{B}$, carbon $_6\text{C}$, nitrogen $_7\text{N}$, oxygen $_8\text{O}$, fluorine $_9\text{F}$, neon $_{10}\text{Ne}$.*
- (3) *Sodium $_{11}\text{Na}$, magnesium $_{12}\text{Mg}$, aluminium $_{13}\text{Al}$, silicon $_{14}\text{Si}$, phosphorus $_{15}\text{P}$, sulfur $_{16}\text{S}$, chlorine $_{17}\text{Cl}$, argon $_{18}\text{Ar}$.*
- (4) *Potassium $_{19}\text{K}$, calcium $_{20}\text{Ca}$, scandium $_{21}\text{Sc}$, titanium $_{22}\text{Ti}$, vanadium $_{23}\text{V}$, chromium $_{24}\text{Cr}$, manganese $_{25}\text{Mn}$, iron $_{26}\text{Fe}$, cobalt $_{27}\text{Co}$.*
- (5) *Nickel $_{28}\text{Ni}$, copper $_{29}\text{Cu}$, zinc $_{30}\text{Zn}$, gallium $_{31}\text{Ga}$, germanium $_{32}\text{Ge}$, arsenic $_{33}\text{As}$, selenium $_{34}\text{Se}$, bromine $_{35}\text{Br}$, krypton $_{36}\text{Kr}$.*

Observe that all names fit with the abbreviations, expect for sodium $_{11}\text{Na}$, coming from the Latin natrium, potassium $_{19}\text{K}$, coming from the Latin kalium, iron $_{26}\text{Fe}$ coming from the Latin ferrum, and also copper $_{29}\text{Cu}$, coming from the Latin cuprum.

In what regards the elements heavier than krypton $_{36}\text{Kr}$, it is heartbreaking to sort them out, but as a useful complement to the above list, we have:

FACT 1.36. *Remarkable elements heavier than krypton $_{36}\text{Kr}$ include:*

- (1) *Noble gases: xenon $_{54}\text{Xe}$, radon $_{86}\text{Rn}$.*
- (2) *Noble metals: silver $_{47}\text{Ag}$, iridium $_{77}\text{Ir}$, platinum $_{78}\text{Pt}$, gold $_{47}\text{Au}$.*
- (3) *Heavy metals: mercury $_{80}\text{Hg}$, lead $_{82}\text{Pb}$.*
- (4) *Radioactive: polonium $_{84}\text{Po}$, radium $_{88}\text{Ra}$, uranium $_{92}\text{U}$, plutonium $_{94}\text{Pu}$.*
- (5) *Miscellaneous: rubidium $_{37}\text{Rb}$, strontium $_{38}\text{Sr}$, molybdenum $_{42}\text{Mo}$, technetium $_{43}\text{Tc}$, cadmium $_{48}\text{Cd}$, tin $_{50}\text{Sn}$, iodine $_{53}\text{I}$, caesium $_{55}\text{Cs}$, tungsten $_{74}\text{Tu}$, bismuth $_{83}\text{Bi}$, francium $_{87}\text{Fr}$, americium $_{95}\text{Am}$.*

Here the abbreviations not fitting with English names come from the Latin or sometimes Greek argentum $_{47}\text{Ag}$, aurum $_{47}\text{Au}$, hydrargyrum $_{80}\text{Hg}$, plumbum $_{82}\text{Pb}$ and stannum $_{50}\text{Sn}$. The noble gases in (1) normally include oganesson $_{118}\text{Og}$ as well. The noble metals in (2) are something subjective. There are of course plenty of other heavy metals (3), or

radioactive elements (4). As for the list in (5), this is something subjective, basically a mixture of well-known metals used in engineering, and some well-known bad guys in the context of nuclear fallout. Technetium ${}_{43}\text{Tc}$ is a bizarre element, human-made.

But let us not forget about quantum mechanics, and what we wanted to do, namely discuss electron structure. As a first observation, we have:

FACT 1.37. *Any $Z = 1, \dots, 118$ corresponds to a unique element, having Z protons in the core, and Z electrons around it. This element might come with isotopes, depending on the number of neutrons in the core, can be in ground state or excited states, can get ionized, and so on, but all there versions are “family”, and the element is unique.*

This fact is something which might look very natural, with no need for explanation for it, but after some thinking, is this really that natural. And the answer here is that no, if you don't know quantum mechanics, and yes, if you know some, as we do:

(1) For the purposes of our question, we can assume that we are in the context of Problem 1.32, and with the Coulomb repulsions between electrons ignored.

(2) But then, we are a bit in the same situation as in Fact 1.33, and the analysis there, based on hydrogen theory modified via $e \rightarrow Ze$, carries over.

(3) And so, the Z electrons will arrange on various energy levels, subject to Pauli exclusion, as to occupy a state of lowest possible energy, so the solution is unique.

In fact, we can now understand the electron structure of the various elements, and also how the periodic table is exactly made, the conclusions here being as follows:

FACT 1.38. *For the element having atomic number Z , the electrons will occupy successively the various positions with quantum numbers $n, l, m \in \mathbb{N}$ and spin $s = \pm 1/2$, such as the total binding energy to be minimal. In practice, the period $1, \dots, 7$ corresponds to the highest n occupied, and the group $1, \dots, 18$ comes from l, m, s .*

This is of course something very basic, and there is a detailed analysis to be done afterwards, for $Z = 1, \dots, 118$. For the elements up to krypton ${}_{36}\text{Kr}$, the list of electron configurations can be found for instance in Feynman [32] or Griffiths [41].

With this in hand, we can even start doing some chemistry. There is an enormous quantity of things that can be said here, the simplest of which being:

THEOREM 1.39. *The group 18 elements, helium ${}_{2}\text{He}$, neon ${}_{10}\text{Ne}$, argon ${}_{18}\text{Ar}$, krypton ${}_{36}\text{Kr}$, xenon ${}_{54}\text{Xe}$ and radon ${}_{86}\text{Rn}$, called noble gases, are allergic to chemistry.*

PROOF. This follows from the above discussion, because the group 18 elements are precisely those with all possible electron positions fully occupied, up to a certain $n \in \mathbb{N}$, which makes them very unfriendly to any chemistry proposition from the outside. By the way, oganesson ${}_{118}\text{Og}$ is normally part of this group too, but since this element has only been created and observed for a tiny fraction of a second, who really knows, and by the standard scientific etiquette, in the lack of experiments, no comment about it. \square

1e. Exercises

Exercises:

EXERCISE 1.40.

EXERCISE 1.41.

EXERCISE 1.42.

EXERCISE 1.43.

EXERCISE 1.44.

EXERCISE 1.45.

Bonus exercise.

CHAPTER 2

Basic properties

2a. Solid state

2b. Magnetization

2c. Melting and beyond

2d. Heavy metals

2e. Exercises

Exercises:

EXERCISE 2.1.

EXERCISE 2.2.

EXERCISE 2.3.

EXERCISE 2.4.

EXERCISE 2.5.

EXERCISE 2.6.

Bonus exercise.

CHAPTER 3

Stress, elasticity

3a. Elasticity

3b. Metal bars

3c. Sheet metal

3d. Springs

3e. Exercises

Exercises:

EXERCISE 3.1.

EXERCISE 3.2.

EXERCISE 3.3.

EXERCISE 3.4.

EXERCISE 3.5.

EXERCISE 3.6.

Bonus exercise.

CHAPTER 4

Oxydation

4a. Molecules

4b. Oxygen

4c. Oxydation

4d. Basic compounds

4e. Exercises

Exercises:

EXERCISE 4.1.

EXERCISE 4.2.

EXERCISE 4.3.

EXERCISE 4.4.

EXERCISE 4.5.

EXERCISE 4.6.

Bonus exercise.

Part II

Geology

*Then you took me to St. Louis
Later on, down the river
A freighter said she's been here
But she's gone, boy, she's gone*

CHAPTER 5

Stars, genesis

5a. High energy

Welcome to geology. We will be looking here for metals ores in the Earth crust, that we can extract and exploit afterwards. Which looks like an easy job, you would say, just spot some promising landscapes, featuring interesting rocks, and start digging.

And this is what we will do, more or less, but before that, some physics preliminaries. We would first like to understand where the Earth, and all its metals, come from. And this is no easy question, with the answer to it involving stars, their formation, functioning and collapse, then the formation of planets, the daily life of these planets, and so on.

So, to start with, we first need to have a trip to the outer space, with a look at stars, and their physics. And here, hang on, things will be radically different from what we know from our modest human existence, here on Earth, with almost all numbers involved being 1,000 or 1,000,000 or 1,000,000,000 bigger than those that we are used to.

Which leads us to the question, what to do, as a preliminary, in order to get used to such big numbers, and to extreme physics in general? Good question, and in answer, for everything extreme physics, nothing beats the Einstein $E = mc^2$ formula. So, getting back to what we wanted to do, here is the plan for the present Part II of this book:

PLAN 5.1. *In order to get introduced to geology, we will talk about:*

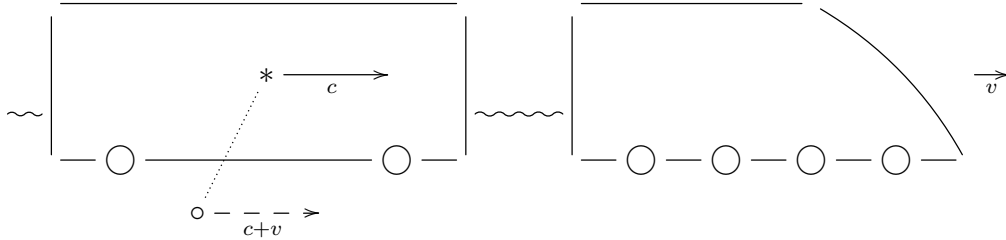
- (1) *Einstein, relativity and $E = mc^2$, as to get into high energy mood.*
- (2) *Main sequence stars, their formation, functioning and collapse.*
- (3) *Planets, their metal core, and the metal available in their crust.*
- (4) *Geology, notably featuring water, and its consequences.*

Getting started now, relativity theory and $E = mc^2$, there is a long story here. As a starting point for our presentation, we will have the following rather philosophical fact:

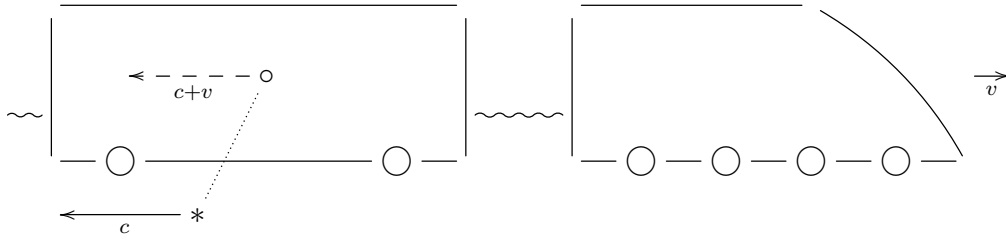
FACT 5.2 (Einstein principles). *The following happen:*

- (1) *Light travels in vacuum at finite speed, $c < \infty$.*
- (2) *This speed c is the same for all inertial observers.*
- (3) *In non-vacuum, the light speed is lower, $v < c$.*
- (4) *Nothing can travel faster than light, $v \not> c$.*

Observe now that all this, and more specifically (2), contradicts classical mechanics. Indeed, assuming that we have a train, running in vacuum at speed $v > 0$, and someone on board lights a flashlight $*$ towards the locomotive, then an observer \circ on the ground will see the light travelling at speed $c + v > c$, which is a contradiction:



Equivalently, with the same train running, in vacuum at speed $v > 0$, if the observer on the ground lights a flashlight $*$ towards the back of the train, then viewed from the train, that light will travel at speed $c + v > c$, which is a contradiction again:



Summarizing, Fact 5.2 implies $c + v = c$, so contradicts classical mechanics, which therefore needs a fix. But the fix in 1D is straightforward, as follows:

THEOREM 5.3. *If we sum the speeds according to the Einstein formula*

$$u +_e v = \frac{u + v}{1 + uv}$$

in $c = 1$ units, then the Galileo formula still holds, approximately, for low speeds

$$u +_e v \simeq u + v$$

and if we have $u = 1$ or $v = 1$, the resulting sum is $u +_e v = 1$.

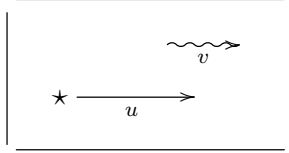
PROOF. All this is self-explanatory, and clear from definitions, and with the Einstein formula of $u +_e v$ itself being just the obvious solution to our $c + v = c$ puzzle. To be more precise, if we plug in $u = 1$ in the above summation formula, we obtain as result:

$$1 +_e v = \frac{1 + v}{1 + v} = 1$$

And the same happens with $v = 1$. Thus, we are led to the above conclusions. \square

All the above looks quite good, theoretically speaking, and also, importantly, agrees with various findings from experimental physics, and notably with:

EXPERIMENT 5.4 (Fizeau, 1851). Assume that light moves through a liquid at speed $u < c$. Then, when this liquid moves through a tube at speed $v > 0$,



the observed speed of light is not the Galilean $u +_g v = u + v$, but rather

$$u +_f v = u + v \left(1 - \frac{1}{n^2} \right)$$

where $n = c/u$ is the index of refraction of the liquid.

You must agree with me that this looks very good, especially in what regards the first part, with the observed speed by Fizeau being not the Galilean one, $u +_f v \neq u +_g v$, but then with the second part too, with Fizeau's sum $u +_f v$ looking quite similar to the Einstein sum $u +_e v$. So, let us do now the math, and compare what Fizeau and Einstein say. The result here, which is certainly a success, if you are a bit familiar with the difficulties of experimental physics, say via daily cooking at home, is as follows:

THEOREM 5.5. The Fizeau speed summation, which in $c = 1$ units is

$$u +_f v = u + v - u^2 v \simeq (u + v)(1 - uv)$$

is compatible with the Einstein speed summation, which in $c = 1$ units is

$$u +_e v = \frac{u + v}{1 + uv} \simeq (u + v)(1 - uv)$$

with the approximations coming from $u \gg v$, and from $1/(1 + x) \simeq 1 - x$.

PROOF. This is something rather self-explanatory, but let us work out the details. In $c = 1$ units the index of refraction of the liquid is $n = 1/u$, and we have:

$$\begin{aligned} u +_f v &= u + v \left(1 - \frac{1}{(1/u)^2} \right) \\ &= u + v(1 - u^2) \\ &= u + v - u^2 v \\ &\simeq u + v - u^2 v - uv^2 \\ &= (u + v)(1 - uv) \end{aligned}$$

To be more precise here, we have used, for the approximation at the end:

$$v \ll u \implies uv^2 \ll u, v, u^2 v$$

As for the processing of the Einstein formula, this simply uses $1/(1 + x) \simeq 1 - x$. \square

Now back to theory, based on what we have so far, namely Theorem 5.3. we can further build on this, with the following quite surprising conclusions:

THEOREM 5.6. *Time and length are subject to Lorentz dilation and contraction*

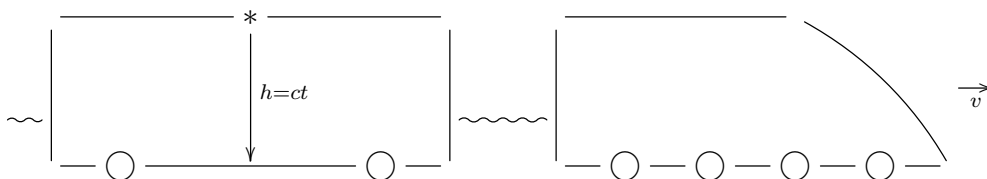
$$t \rightarrow \gamma t \quad , \quad L \rightarrow L/\gamma$$

where the number $\gamma \geq 1$, called Lorentz factor, is given by the formula

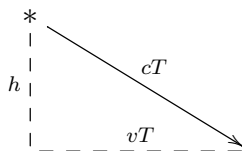
$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

with v being the moving speed, at which time is measured.

PROOF. We use our favorite train. In order to compute the height h of the train, the passenger switches on the ceiling light bulb, measures the time t that the light needs to hit the floor, by travelling at speed c , and concludes that the train height is $h = ct$:



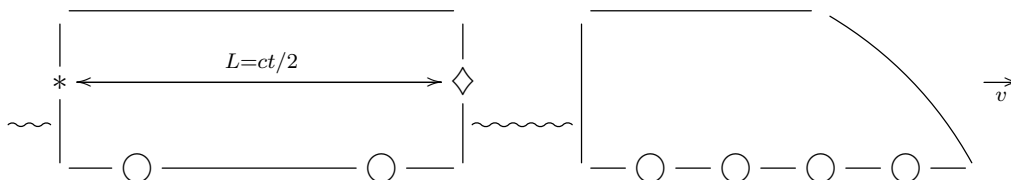
On the other hand, an observer on the ground will see here something different, namely a right triangle, with on the vertical the height of the train h , on the horizontal the distance vT that the train has travelled, and on the hypotenuse the distance cT that light has travelled, with T being the duration of the event, according to his watch:



But this gives, via Pythagoras and some calculus, the time dilation formula:

$$T = \gamma t$$

Regarding now length, imagine that the passenger wants to measure the length L of the car. For this purpose he switches on the light bulb, now at the rear of the car, and measures the time t needed for the light to reach the front of the car, and get reflected back by a mirror installed there, according to the following scheme:



Now viewed from the ground, the duration of the event is $T = T_1 + T_2$, where $T_1 > T_2$ are respectively the time needed for the light to travel forward, among others for beating v , and the time for the light to travel back, helped this time by v . More precisely, if l denotes the length of the train car viewed from the ground, the formula of T is:

$$T = \frac{l}{c-v} + \frac{l}{c+v} = \frac{2lc}{c^2 - v^2}$$

With this data, the formula $T = \gamma t$ of time dilation established before reads:

$$l = \frac{L}{\gamma}$$

Thus, we are led to the conclusions in the statement. □

As a main consequence of the above, beautiful as they come, we have:

THEOREM 5.7. *In the context of a relativistic object moving with speed v along the x axis, the frame change is given by the Lorentz transformation*

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma(t - vx/c^2)$$

with $\gamma = 1/\sqrt{1 - v^2/c^2}$ being as usual the Lorentz factor.

PROOF. We know that, with respect to the non-relativistic formulae, x is subject to the Lorentz dilation by γ , and so we obtain, as desired:

$$x' = \gamma(x - vt)$$

Regarding y, z , these are obviously unchanged, so done with these too. Finally, regarding time t , we can use here the reverse Lorentz transformation, given by:

$$x = \gamma(x' + vt')$$

$$y = y'$$

$$z = z'$$

By using the formula of x' we can compute t' , and we obtain the following formula:

$$t' = \frac{x - \gamma x'}{\gamma v} = \frac{x - \gamma^2(x - vt)}{\gamma v} = \gamma \left(t - \frac{vx}{c^2} \right)$$

We are therefore led to the conclusion in the statement. □

Before getting further with more relativity, let us record as well the following key result, in relation with electromagnetism, due to Lorentz:

THEOREM 5.8. *The Maxwell equations for electromagnetism, namely*

$$\langle \nabla, E \rangle = \frac{\rho}{\varepsilon_0} \quad , \quad \langle \nabla, B \rangle = 0$$

$$\nabla \times E = -\dot{B} \quad , \quad \nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \dot{E}$$

are invariant under Lorentz transformations.

PROOF. Consider an electromagnetic field (E, B) . This is altered by a Lorentz transformation into a field (E', B') , the equations for E' being as follows:

$$E'_x = E_x$$

$$E'_y = \gamma(E_y - vB_z)$$

$$E'_z = \gamma(E_z + vB_y)$$

As for the equations of B' , these are quite similar, as follows:

$$B'_x = B_x$$

$$B'_y = \gamma\left(B_y + \frac{v}{c^2}E_z\right)$$

$$B'_z = \gamma\left(B_z - \frac{v}{c^2}E_y\right)$$

In order to do the math, consider the following matrices, with $\beta = v/c$ as usual:

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \gamma & 0 \\ 0 & 0 & \gamma \end{pmatrix} \quad , \quad M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\beta\gamma \\ 0 & \beta\gamma & 0 \end{pmatrix}$$

In terms of these matrices, the formulae for the new field (E', B') read:

$$E' = DE + cMB \quad , \quad B' = DB - \frac{M}{c}E$$

But this is already not that bad, and starting from these formulae, it is possible to prove that (E', B') satisfies as well the Maxwell equations, as desired. \square

Now back to relativity, in relation with mass and momentum, we have the following key fact, which is something quite natural, and is of course, in tune with experiments:

FACT 5.9. *When defining the relativistic mass of an object of rest mass $m > 0$, moving at speed v , by the formula*

$$M = \gamma m \quad : \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

this relativistic mass M , and the corresponding relativistic momentum $P = Mv$, are both conserved during collisions.

With this in hand, we can now discuss kinetic energy. You have certainly heard of the formula $E = mc^2$, which might actually well be on your T-shirt, now as you read this book, and in this case here is the explanation for it, in relation with the above:

THEOREM 5.10. *The relativistic energy of an object of rest mass $m > 0$,*

$$\mathcal{E} = Mc^2 \quad : \quad M = \gamma m$$

which is conserved, as being a multiple of M , can be written as $\mathcal{E} = E + T$, with

$$E = mc^2$$

being its $v = 0$ component, called rest energy of m , and with

$$T = (1 - \gamma)mc^2 \simeq \frac{mv^2}{2}$$

being called relativistic kinetic energy of m .

PROOF. All this is a bit abstract, coming from Fact 5.9, as follows:

(1) Given an object of rest mass $m > 0$, consider its relativistic mass $M = \gamma m$, as appearing in Fact 5.9, and then consider the following quantity:

$$\mathcal{E} = Mc^2$$

We know from Fact 5.9 that the relativistic mass M is conserved, so $\mathcal{E} = Mc^2$ is conserved too. In view of this, it makes somehow sense to call \mathcal{E} energy. There is of course no clear reason for doing that, but let's just do it, and we'll understand later.

(2) Let us compute \mathcal{E} . This quantity is by definition given by:

$$\mathcal{E} = Mc^2 = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$

Since $1/\sqrt{1-x} \simeq 1 + x/2$ for x small, by calculus, we obtain, for v small:

$$\mathcal{E} \simeq mc^2 \left(1 + \frac{v^2}{2c^2} \right) = mc^2 + \frac{mv^2}{2}$$

And, good news here, we recognize at right the kinetic energy of m .

(3) But this leads to the conclusions in the statement. Indeed, we are certainly dealing with some sort of energies here, and so calling the above quantity \mathcal{E} relativistic energy is legitimate, and calling $E = mc^2$ rest energy is legitimate too. Finally, the difference between these two energies $T = \mathcal{E} - E$ follows to be given by:

$$T = (1 - \gamma)mc^2 \simeq \frac{mv^2}{2}$$

Thus, calling T relativistic kinetic energy is legitimate too, and we are done. \square

Summarizing, we have managed to understand what $E = mc^2$ is about. Now let us try to understand the numerics, and consequences. At the terrestrial level, we have:

THEOREM 5.11. *An atomic bomb based on a glass of water releasing all its $E = mc^2$ energy is equivalent to the Giza Pyramid hitting you at 30 km/s.*

PROOF. When converting $E = mc^2$ into kinetic energy of a body M , the formula is:

$$mc^2 = \frac{Mv^2}{2} \implies v = \sqrt{\frac{2m}{M}} c$$

In our case the glass of water m , glass included, is about 300 grams, and the Giza Pyramid M is about 6 million tons. Thus the impact speed is:

$$v = \sqrt{\frac{2 \times 3 \times 10^{-1}}{6 \times 10^9}} c = \frac{c}{10^5} = 30,000 \text{ m/s}$$

As already mentioned, do not worry about this. Certain atomic bombs, based on other materials, can indeed be constructed. But the glass of water, as long as it stays far away from a big source of energy, of stellar or atomic type, will certainly not explode. \square

5b. Atomic fusion

5c. Main sequence stars

5d. Star collapse

5e. Exercises

Exercises:

EXERCISE 5.12.

EXERCISE 5.13.

EXERCISE 5.14.

EXERCISE 5.15.

EXERCISE 5.16.

EXERCISE 5.17.

Bonus exercise.

CHAPTER 6

Planet Earth

6a. Dust, planets

6b. Metallic core

6c. Basic geology

6d. Water effects

6e. Exercises

Exercises:

EXERCISE 6.1.

EXERCISE 6.2.

EXERCISE 6.3.

EXERCISE 6.4.

EXERCISE 6.5.

EXERCISE 6.6.

Bonus exercise.

CHAPTER 7

Basic compounds

7a. Basic compounds

7b.

7c.

7d.

7e. Exercises

Exercises:

EXERCISE 7.1.

EXERCISE 7.2.

EXERCISE 7.3.

EXERCISE 7.4.

EXERCISE 7.5.

EXERCISE 7.6.

Bonus exercise.

CHAPTER 8

Meteorites

8a. Meteorites

8b.

8c.

8d.

8e. Exercises

Exercises:

EXERCISE 8.1.

EXERCISE 8.2.

EXERCISE 8.3.

EXERCISE 8.4.

EXERCISE 8.5.

EXERCISE 8.6.

Bonus exercise.

Part III

Metallurgy

*If I were a carpenter
And you were a lady
Would you marry me anyway
Would you have my baby*

CHAPTER 9

Stone Age

9a. Stone Age

9b.

9c.

9d.

9e. Exercises

Exercises:

EXERCISE 9.1.

EXERCISE 9.2.

EXERCISE 9.3.

EXERCISE 9.4.

EXERCISE 9.5.

EXERCISE 9.6.

Bonus exercise.

CHAPTER 10

Silver and gold

10a. Silver and gold

10b.

10c.

10d.

10e. Exercises

Exercises:

EXERCISE 10.1.

EXERCISE 10.2.

EXERCISE 10.3.

EXERCISE 10.4.

EXERCISE 10.5.

EXERCISE 10.6.

Bonus exercise.

CHAPTER 11

Cooper, bronze

11a. Cooper, bronze

11b.

11c.

11d.

11e. Exercises

Exercises:

EXERCISE 11.1.

EXERCISE 11.2.

EXERCISE 11.3.

EXERCISE 11.4.

EXERCISE 11.5.

EXERCISE 11.6.

Bonus exercise.

CHAPTER 12

Iron and steel

12a. Iron and steel

12b.

12c.

12d.

12e. Exercises

Exercises:

EXERCISE 12.1.

EXERCISE 12.2.

EXERCISE 12.3.

EXERCISE 12.4.

EXERCISE 12.5.

EXERCISE 12.6.

Bonus exercise.

Part IV

Steel

On a Monday, I was arrested
On a Tuesday, they locked me in the jail
On a Wednesday, my trial was attested
On a Thursday, they said guilty and the judge's gavel fell

CHAPTER 13

Iron, revised

13a. Iron, revised

13b.

13c.

13d.

13e. Exercises

Exercises:

EXERCISE 13.1.

EXERCISE 13.2.

EXERCISE 13.3.

EXERCISE 13.4.

EXERCISE 13.5.

EXERCISE 13.6.

Bonus exercise.

CHAPTER 14

Steel, alloys

14a. Steel, alloys

14b.

14c.

14d.

14e. Exercises

Exercises:

EXERCISE 14.1.

EXERCISE 14.2.

EXERCISE 14.3.

EXERCISE 14.4.

EXERCISE 14.5.

EXERCISE 14.6.

Bonus exercise.

CHAPTER 15

Carbon steel

15a. Carbon steel

15b.

15c.

15d.

15e. Exercises

Exercises:

EXERCISE 15.1.

EXERCISE 15.2.

EXERCISE 15.3.

EXERCISE 15.4.

EXERCISE 15.5.

EXERCISE 15.6.

Bonus exercise.

CHAPTER 16

Axes and knives

16a. Axes and knives

16b.

16c.

16d.

16e. Exercises

Congratulations for having read this book, and no exercises for this final chapter.

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Index

- accelerating charge, 13
- amplitude, 13
- angular frequency, 13
- atom, 21
- atomic bomb, 48
- atomic number, 28

- Bohr formula, 24

- collision, 46
- colors, 15
- Coulomb potential, 23

- decelerating charge, 13

- electric permittivity, 16
- electrodynamics, 12
- electromagnetic radiation, 13
- electromagnetic wave, 13
- electron, 22

- Fizeau addition, 42
- Fizeau experiment, 42
- Fourier analysis, 13
- Fourier transform, 17
- free space, 13
- frequency, 15

- gamma ray, 15

- Hamiltonian, 24, 28
- helium, 28
- homogeneous medium, 16

- index of refraction, 42
- IR, 15

- Laplace operator, 13

- light, 13
- light through liquid, 42
- Lorentz factor, 45
- Lorentz transformation, 45

- magnetic permeability, 16
- Maxwell equations, 12, 13, 16, 45
- microwave, 15
- monochromatic, 14

- noble gases, 31

- observable, 23

- Pauli exclusion, 29
- phase, 13
- Planck constant, 22
- Planck formula, 27
- polarization, 15
- polarized light, 15
- prism, 17

- radio wave, 15
- reflection angle, 17
- refraction, 17
- refraction angle, 17
- refraction index, 16
- relativistic energy, 47
- relativistic kinetic energy, 47
- relativistic mass, 46
- relativistic momentum, 46
- relativity, 45
- rest energy, 47
- rest mass, 46, 47
- Rydberg formula, 27

- Schrödinger equation, 22, 28

separation of variables, 23

Snell law, 17

spectroscopy, 17

speed of light, 13, 16

time-independent equation, 23

time-independent potential, 23

total energy, 24

UV, 15

wave, 13

wave equation, 13

wave function, 22

wave number, 13

wave packet, 13

wavelength, 15

X ray, 15