Mechanics of vertebrates

Teo Banica

Department of Mathematics, University of Cergy-Pontoise, F-95000 Cergy-Pontoise, France. teo.banica@gmail.com

2010 *Mathematics Subject Classification.* 92C10

Key words and phrases. Biomechanics, Vertebrates

Abstract. This is an introduction to the mechanics of vertebrates, by insisting on abstract modeling, and mathematical aspects. We first discuss the physics of life in general, with a look at the various laws of physics and chemistry, and the constants and other data involved, and which types of life forms they allow. Then we discuss the origins, story and mechanical functioning of the vertebrates, with various mathematical models for the skeleton, muscles, and energy system. We then go on a more detailed discussion regarding locomotion and jumps, and with a look into flying too. We end with a similar discussion, even more specialized, regarding the animal bites.

Preface

You certainly know how to walk, run, jump and swim, but what is the exact mechanics behind that? You would say, as an approximation, that we are some sort of robots using food instead of electricity, via a remarkable digestive system, then blood, and skeleton and muscles, as we all learned in school. And that is for sure the good answer, but when it comes to details, how all this exactly works, mechanically speaking, and what limits can be achieved, by you, or by humans in general, and then what about other vertebrates, what they can do and what they can't, when compared to us, and so on, things become quite complicated and fascinating. And with this being the subject of biomechanics.

The best introduction to biomechanics remains doing some individual sports, such as running, or cycling, or swimming, or weight lifting, with the aim of knowing, and then improving your limits. The knowledge that you can acquire in this way is golden, every little misfortune on the way will push you into learning more and more about anatomy, and metabolism, and many more, and cannot replace any theoretical learning. So, in the hope that you are indeed doing some sports, and know what I'm talking about.

At a more advanced level now, making a science out of biomechanics, things quickly turn complex. Who has not wished, in the sports context, to be much lighter, and with stronger muscles, or why not walking on 4 legs, or having a tail, or why not a pair of wings, or perhaps no legs at all and a strong tail instead, depending on the situation. In short, the biomechanics questions quickly move us into the general animal kingdom, or perhaps just into vertebrates, with this science being best understood in that context.

The present book is an introduction to this, mechanics of the vertebrates, with the basic needed anatomy explained, and by insisting on abstract modeling questions, and on mathematical aspects. The book is organized in 4 parts, the plan being as follows:

I - As a philosophical prelude to biomechanics, we will first discuss the physics of life in general, with a look at the various laws of physics and chemistry, and the constants and other data involved, and which types of life forms they allow.

II - Then we will discuss the origins, story and mechanical functioning of the vertebrates, with some reasonably advanced biology and anatomy explanations, and with various mathematical models for the skeleton, muscles, and energy system.

III - We will then go on a more detailed discussion regarding locomotion, speed and jumps, and with a look into other forms of locomotion, like swimming and flying, too. Quite fascinating here is the snake, but big cats will be among our favorites, too.

IV - We will end with a similar discussion, regarding this time the animal bites. Here the crown for speed goes to several amazing fishes, but we will mostly insist on more familiar animals, such as bears, cats again, hyenas, and of course, the crocodile.

It is a pleasure to thank everyone having helped me with learning some of this material, including various colleagues, friends, also a bike dealer, once a doctor, and many more. Many thanks as well to my cats, normally my daytime job is quantum physicist, but with them around, it is difficult not to be distracted by muscles, speed and biomechanics.

Contents

Part I

Physics of life

Hey you
Out there in the cold
Getting lonely, getting old
Can you feel me?

CHAPTER 1

Physics constants

1a.

1b.

1c.

1d.

1e. Exercises

Exercises:

Exercise 1.1.

Exercise 1.2.

Exercise 1.3.

Exercise 1.4.

Exercise 1.5.

Exercise 1.6.

Exercise 1.7.

Exercise 1.8.

Bonus exercise.

CHAPTER 2

Earth data

2a.

2b.

2c.

2d.

2e. Exercises

Exercises:

Exercise 2.1.

Exercise 2.2.

Exercise 2.3.

Exercise 2.4.

Exercise 2.5.

Exercise 2.6.

Exercise 2.7.

Exercise 2.8.

Bonus exercise.

CHAPTER 3

Life, limits

3a.

3b.

3c.

3d.

3e. Exercises

Exercises:

Exercise 3.1.

Exercise 3.2.

Exercise 3.3.

Exercise 3.4.

Exercise 3.5.

Exercise 3.6.

Exercise 3.7.

Exercise 3.8.

Bonus exercise.

CHAPTER 4

Small vs big

4a.

4b.

4c.

4d.

4e. Exercises

Exercises:

Exercise 4.1.

Exercise 4.2.

Exercise 4.3.

Exercise 4.4.

Exercise 4.5.

Exercise 4.6.

Exercise 4.7.

Exercise 4.8.

Bonus exercise.

Part II

Vertebrates

I got the eye of the tiger, a fighter
Dancing through the fire
Cause I am a champion
And you're gonna hear me roar

CHAPTER 5

Origins, story

5a.

5b.

5c.

5d.

5e. Exercises

Exercises:

Exercise 5.1.

Exercise 5.2.

Exercise 5.3.

Exercise 5.4.

Exercise 5.5.

Exercise 5.6.

Exercise 5.7.

Exercise 5.8.

Bonus exercise.

CHAPTER 6

The skeleton

6a.

6b.

6c.

6d.

6e. Exercises

Exercises:

Exercise 6.1.

Exercise 6.2.

Exercise 6.3.

Exercise 6.4.

Exercise 6.5.

Exercise 6.6.

Exercise 6.7.

Exercise 6.8.

Bonus exercise.

CHAPTER 7

Muscles and more

7a.

7b.

7c.

7d.

7e. Exercises

Exercises:

Exercise 7.1.

Exercise 7.2.

Exercise 7.3.

Exercise 7.4.

Exercise 7.5.

Exercise 7.6.

Exercise 7.7.

Exercise 7.8.

Bonus exercise.

CHAPTER 8

Blood, energy

8a.

8b.

8c.

8d.

8e. Exercises

Exercises:

Exercise 8.1.

Exercise 8.2.

Exercise 8.3.

Exercise 8.4.

Exercise 8.5.

Exercise 8.6.

Exercise 8.7.

Exercise 8.8.

Bonus exercise.

Part III

Speed, jumps

If you think you're getting away
I will prove you wrong
I'll take you all the way
Stay another song

CHAPTER 9

Locomotion, speed

9a.

9b.

9c.

9d.

9e. Exercises

Exercises:

Exercise 9.1.

Exercise 9.2.

Exercise 9.3.

Exercise 9.4.

Exercise 9.5.

Exercise 9.6.

Exercise 9.7.

Exercise 9.8.

Bonus exercise.

CHAPTER 10

The snake

10a.

10b.

10c.

10d.

10e. Exercises

Exercises:

Exercise 10.1.

Exercise 10.2.

Exercise 10.3.

Exercise 10.4.

Exercise 10.5.

Exercise 10.6.

Exercise 10.7.

Exercise 10.8.

Bonus exercise.

CHAPTER 11

Jumps, big cats

11a.

11b.

11c.

11d.

11e. Exercises

Exercises:

Exercise 11.1.

Exercise 11.2.

Exercise 11.3.

Exercise 11.4.

Exercise 11.5.

Exercise 11.6.

Exercise 11.7.

Exercise 11.8.

Bonus exercise.

CHAPTER 12

A word on flying

12a.

12b.

12c.

12d.

12e. Exercises

Exercises:

Exercise 12.1.

Exercise 12.2.

Exercise 12.3.

Exercise 12.4.

Exercise 12.5.

Exercise 12.6.

Exercise 12.7.

Exercise 12.8.

Bonus exercise.

Part IV

Animal bites

If you didn't care what happened to me
And I didn't care for you
We would zigzag our way through the boredom and pain
Occasionally glancing up through the rain

CHAPTER 13

Animal bites

13a.

13b.

13c.

13d.

13e. Exercises

Exercises:

Exercise 13.1.

Exercise 13.2.

Exercise 13.3.

Exercise 13.4.

Exercise 13.5.

Exercise 13.6.

Exercise 13.7.

Exercise 13.8.

Bonus exercise.

CHAPTER 14

Cats and hyenas

14a.

14b.

14c.

14d.

14e. Exercises

Exercises:

Exercise 14.1.

Exercise 14.2.

Exercise 14.3.

Exercise 14.4.

Exercise 14.5.

Exercise 14.6.

Exercise 14.7.

Exercise 14.8.

Bonus exercise.

CHAPTER 15

Big fellows

15a.

15b.

15c.

15d.

15e. Exercises

Exercises:

Exercise 15.1.

Exercise 15.2.

Exercise 15.3.

Exercise 15.4.

Exercise 15.5.

Exercise 15.6.

Exercise 15.7.

Exercise 15.8.

Bonus exercise.

CHAPTER 16

The crocodile

16a.

16b.

16c.

16d.

16e. Exercises

Congratulations for having read this book, and no exercises for this final chapter.

Bibliography

[1] V.I. Arnold, Ordinary differential equations, Springer (1973).

[2] V.I. Arnold, Lectures on partial differential equations, Springer (1997).

[3] V.I. Arnold, Catastrophe theory, Springer (1984).

[4] T. Banica, Calculus and applications (2024).

[5] T. Banica, Introduction to modern physics (2024).

[6] R.J. Baxter, Exactly solved models in statistical mechanics, Academic Press (1982).

[7] S.J. Blundell and K.M. Blundell, Concepts in thermal physics, Oxford Univ. Press (2006).

[8] S.M. Carroll, Spacetime and geometry, Cambridge Univ. Press (2004).

[9] P.A.M. Dirac, Principles of quantum mechanics, Oxford Univ. Press (1930).

[10] S. Dodelson, Modern cosmology, Academic Press (2003).

[11] R. Durrett, Probability: theory and examples, Cambridge Univ. Press (1990).

[12] A. Einstein, Relativity: the special and the general theory, Dover (1916).

[13] L.C. Evans, Partial differential equations, AMS (1998).

[14] E. Fermi, Thermodynamics, Dover (1937).

[15] R.P. Feynman, R.B. Leighton and M. Sands, The Feynman lectures on physics I: mainly mechanics, radiation and heat, Caltech (1963).

[16] R.P. Feynman, R.B. Leighton and M. Sands, The Feynman lectures on physics II: mainly electromagnetism and matter, Caltech (1964).

[17] R.P. Feynman, R.B. Leighton and M. Sands, The Feynman lectures on physics III: quantum mechanics, Caltech (1966).

[18] H. Goldstein, C. Safko and J. Poole, Classical mechanics, Addison-Wesley (1980).

[19] D.J. Griffiths, Introduction to electrodynamics, Cambridge Univ. Press (2017).

[20] D.J. Griffiths and D.F. Schroeter, Introduction to quantum mechanics, Cambridge Univ. Press (2018).

[21] D.J. Griffiths, Introduction to elementary particles, Wiley (2020).

[22] K. Huang, Introduction to statistical physics, CRC Press (2001).

[23] K. Huang, Fundamental forces of nature, World Scientific (2007).

[24] T. Kibble and F.H. Berkshire, Classical mechanics, Imperial College Press (1966).

[25] C. Kittel, Introduction to solid state physics, Wiley (1953).

[26] M. Kumar, Quantum: Einstein, Bohr, and the great debate about the nature of reality, Norton (2009).

[27] T. Lancaster and K.M. Blundell, Quantum field theory for the gifted amateur, Oxford Univ. Press (2014).

[28] P. Lax, Functional analysis, Wiley (2002).

[29] P. Lax and M.S. Terrell, Calculus with applications, Springer (2013).

[30] P. Lax and M.S. Terrell, Multivariable calculus with applications, Springer (2018).

[31] M.A. Nielsen and I.L. Chuang, Quantum computation and quantum information, Cambridge Univ. Press (2000).

[32] R.K. Pathria and and P.D. Beale, Statistical mechanics, Elsevier (1972).

[33] B.M. Peterson and B. Ryden, Foundations of astrophysics, Cambridge Univ. Press (2010).

[34] W. Rudin, Principles of mathematical analysis, McGraw-Hill (1964).

[35] W. Rudin, Real and complex analysis, McGraw-Hill (1966).

[36] W. Rudin, Functional analysis, McGraw-Hill (1973).

[37] B. Ryden, Introduction to cosmology, Cambridge Univ. Press (2002).

[38] D.V. Schroeder, An introduction to thermal physics, Oxford Univ. Press (1999).

[39] R. Shankar, Fundamentals of physics I: mechanics, relativity, and thermodynamics, Yale Univ. Press (2014).

[40] R. Shankar, Fundamentals of physics II: electromagnetism, optics, and quantum mechanics, Yale Univ. Press (2016).

[41] A.M. Steane, Thermodynamics, Oxford Univ. Press (2016).

[42] J.R. Taylor, Classical mechanics, Univ. Science Books (2003).

[43] J. von Neumann, Mathematical foundations of quantum mechanics, Princeton Univ. Press (1955).

[44] J. von Neumann and O. Morgenstern, Theory of games and economic behavior, Princeton Univ. Press (1944).

[45] S. Weinberg, Foundations of modern physics, Cambridge Univ. Press (2011).

[46] S. Weinberg, Lectures on quantum mechanics, Cambridge Univ. Press (2012).

[47] S. Weinberg, Lectures on astrophysics, Cambridge Univ. Press (2019).

[48] H. Weyl, The theory of groups and quantum mechanics, Princeton Univ. Press (1931).

[49] H. Weyl, The classical groups: their invariants and representations, Princeton Univ. Press (1939).

[50] H. Weyl, Space, time, matter, Princeton Univ. Press (1918).