# Nature and evolution

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ABSTRACT. This is a general introduction to evolution, mainly biology, but with some other illustrations too, written for science students. We first discuss the basics of life and evolution, starting from the very fundamentals, namely atoms and molecules. Then we go on a more technical discussion, regarding the fine evolution mechanisms for our human species. We then discuss other basic examples of evolutionary processes, mostly met in hard science, and going beyond biology. Finally, we have a look at how evolution theory is perceived, positively or negatively, and with acceptation or denial, inside our modern human civilization, and in particular among academics and scientists.

#### Preface

Are we humans superior to the other animals? In fact, are we animals at all, or something else, of more divine nature? How shall we behave, with respect to the general evolution rule for the animal kingdom, shall we comply to that, with awe and pride, or on the opposite, try to fight it? How shall we behave, with respect to the various animal species? What about our organization, as humans, are we made to be rather solitary, as bears, or to live in small groups, as lions, or in large groups, as ants? And so on.

These are all good questions, as old as mankind, and perhaps as old as the animal kingdom too, because right now, as I type and as you read, a horse for instance, once done with his work and meals, might be well thinking at these questions too.

There is no easy, commonly accepted answer to the above questions, at least in our present, modern times. Long time ago, in the golden age of humanity, meaning Stone Age, things were quite simple and luminous. Animals are we, living in small groups, and hunting this and that smaller animals, for food, and beware of not being hunted ourselves, by lions and such. And whenever someone was dying, young or old, by disease or accident, that was surely not very funny, but this is how this life is made, and life goes on.

With time passing by, things got more complicated than this. Human life became increasingly abstract, and away from nature and its rules, and although modern humans are less and less performant, as animals, they tend to think of themselves as more and more advanced, and being above, and not bound to, the general rules of nature.

There are of course many schools of thought here. Traditionally, the main split was between science and religion, with science being in favor of, well, science, and with religion being in favor of, well, religion. More recently, however, with religion increasingly becoming a personal affair, the split became more subtle, and less obvious to spot. In the academia, for instance, the main split nowadays is between science and humanities, and with this being reminiscent of the original split between science and religion.

Moreover, inside academic science you can surely spot some differences too, with some branches of science being still guided, God thanks, by science, while some other being seemingly more and more involved into building some new sort of religion.

#### PREFACE

Quite complicated all this, and the question is now, shall we further talk about all this, which after all might bring us into quite delicate life and academia questions?

Hell yes, I would say, as scientists at least, we are here for talking about things, and the hotter the topic, and better, this is how science always advances, by looking at hard questions, and certainly not by ignoring them. And this book will be here for that, talking first about evolution, and Darwin, with a broad introduction to the subject, and then going into a long discussion about evolution denial, with the main aim of identifying the evolution deniers, inside our present academic system, and society in general.

Many thanks to my cats, and to the other cats in this world, small or big. And in the hope that these cats will remain as they are, and not get involved into some form of religion. That would be bad news for us humans, with them being smarter than us.

Cergy, January 2025 Teo Banica

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# Part I

# Nature, evolution

Sun shines in the rusty morning Skyline of the Olympus Mons I think about it Sometimes

#### Cells, life

#### 1a. Basic physics

In order to talk about cells and life, which will be the main subject of this book, we must first have a look at general physics, and more vulgar forms of matter, such as elementary particles, bare atoms, and basic molecules. The idea indeed, which is already something a bit evolutionary, and so for interest for us in this book, is that elementary particles team up and form atoms, then atoms team up and form molecules, then basic molecules team up and form organic molecules, then organic molecules team up and form cells, and then cells team up and form life. In short, looks like everything in this world is subject to some general laws of order and evolution, that we would like to understand.

So, physics. At the beginnings of modern physics, as we know it nowadays, was classical mechanics, whose main findings can be summarized as follows:

FACT 1.1 (Classical mechanics). The force of attraction between two bodies of masses  $m_1, m_2$ , having distance d > 0 between them, is given by

$$||F|| = G \cdot \frac{m_1 m_2}{d^2}$$

where  $G = 6.674 \times 10^{-11}$  is a constant. This force alters the trajectory of one body with respect to the other according to the following formula, a being the acceleration:

$$F = ma$$

This trajectory is a curve of degree 2, called conic, which can be an ellipsis, parabola, or hyperbola. However, for 3 or more objects, all this can lead to order, or chaos.

Here you surely know all this, with perhaps some gaps in the mathematics at the end, regarding conics and their classification. As an advice, learn this too from somewhere, this is all beautiful and useful mathematics, going back to the ancient Greeks.

Also, in what regards the last sentence, that is a very short summary of what happens for the N-body problem with  $N \ge 3$ , and as a piece of advice here, have a look at Earth scientific satellites and Lagrange points, in relation with N = 3, and also go on internet for more about N = 3, including weird solutions, all this is very interesting.

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As a piece of advertisement here, the interesting problem at N = 3 is how to position a specialized scientific satellite, deep in space, and away from the dust and radiation of the usual orbits around the Earth, as to stay there, under the joint influence of the gravity of the Sun M and of the Earth m. And there are 5 possible solutions here, called Lagrange points L1-L5, whose positions with respect to M, m are as follows:



Moreover, and here comes another interesting point, L4, L5 are stable, in the sense that a satellite installed there will really stay there, regardless of the various tiny little things that might happen, like an asteroid passing by, while L1, L2, L3 are unstable, in the sense that a satellite installed there will need constant tiny adjustments, in order to really stay there. So, which one would you choose for installing your satellite?

You would probably say L4, L5, but this is precisely the wrong answer, because due to their stability, these points attract a lot of asteroids and space garbage, and our satellite will certainly not perform well there, in that crowd. So, with L4, L5 ruled out, and with L3 ruled out too, being too far, the correct choices are L1, L2. But here, you still need to learn a lot more mechanics, for understanding how to do this, in practice.

Moving now towards electrostatics, this is again something very fundamental, that you probably know well too, the basics here being summarized as follows:

FACT 1.2 (Electrostatics). Ordinary matter is made of electrons -, protons + and neutrons 0, with the number of +, - being roughly equal. We set

$$q = \#\{+\} - \#\{-\}$$

and if  $q \neq 0$ , we call this a charge. Any pair of charges  $q_1, q_2 \in \mathbb{R}$  is then subject to a force as follows, which is attractive if  $q_1q_2 < 0$  and repulsive if  $q_1q_2 > 0$ ,

$$||F|| = K \cdot \frac{|q_1 q_2|}{d^2}$$

where  $K = 8.988 \times 10^9$ . However, unlike in classical mechanics,  $q_1 < 0$  will not spin around  $q_2 > 0$  on an ellipsis, due to magnetism, relativity, and quantum mechanics.

Here you are certainly familiar with the Coulomb law formula in the statement, which is very similar to the Newton law formula from Fact 1.1. This normally suggests that when the force is attractive,  $q_1q_2 < 0$ , the negative charge, say an electron -, will spin around the positive charge, say a proton +, on an ellipsis. But this is well-known to be

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#### 1A. BASIC PHYSICS

wrong, with the solution of this 2-body problem, which corresponds to the hydrogen atom, being far more complicated, due to the numerous reasons mentioned in the statement.

Regarding now electrodynamics, this comes as a continuation of electrostatics, with the aim of fixing some of the obvious bugs there, the basics being as follows:

FACT 1.3 (Electrodynamics). Moving charges produce magnetic fields, and the dynamics of the electric fields E and magnetic fields B is governed by the formulae

$$\langle \nabla, E \rangle = \frac{\rho}{\varepsilon_0} \quad , \quad \langle \nabla, B \rangle = 0$$
  
 $\nabla \times E = -\dot{B} \quad , \quad \nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \dot{E}$ 

called Maxwell equations. Also, accelerating or decelerating charges produce electromagnetic radiation, of various wavelengths, called light, of various colors.

Obviously, we are now into serious science here, with the Maxwell equations being something quite complicated, and the pride of 19th century physics, and still the nightmare of everyone using them. To start with, electrodynamics is the science of moving electrical charges. And the problem is that, unlike in classical mechanics, where the Newton law is good for both the static and the dynamic setting, the Coulomb law, which is actually very similar to the Newton law, does the job when the charges are static, but no longer describes well the situation when the charges are moving.

The problem comes from the fact that moving charges produce magnetism, and with this being visible when putting together two electric wires, which will attract or repel, depending on orientation. Thus, in contrast with classical mechanics, where static or dynamic problems are described by a unique field, the gravitational one, in electrodynamics we have two fields, namely the electric field E, and the magnetic field B.

Fortunately, there is a full set of equations relating the electric field E and the magnetic field B, those found by Maxwell and others, given above. Regarding the math,  $\langle , \rangle$  and  $\times$  are the usual scalar and vector products on  $\mathbb{R}^3$ , the dots denote derivatives with respect to time, and  $\nabla$  is the gradient operator, or space derivative, given by:

$$\nabla = \begin{pmatrix} \frac{d}{dx} \\ \frac{d}{dy} \\ \frac{d}{dz} \end{pmatrix}$$

As for the physics, the first formula is the Gauss law,  $\rho$  being the charge, and  $\varepsilon_0$  being a constant, and with this Gauss law more or less replacing the Coulomb law from electrostatics. The second formula is something basic, and anonymous. The third formula is the Faraday law. As for the fourth formula, this is the Ampère law, as modified by Maxwell, with J being the volume current density, and  $\mu_0$  being a constant.

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Importantly, in addition to what is said in Fact 1.3, it is also known that the constants there  $\mu_0, \varepsilon_0$ , which are electrodynamic quantities, are subject to the following magic formula, due to Biot-Savart, with c = 299792458 m/s being the speed of light:

$$\mu_0\varepsilon_0 = \frac{1}{c^2}$$

In what regards now the last sentence, this is something fundamental too, putting an end to centuries or even millenia of discussions, regarding the nature of light. Speaking light, here is the table coming from Fact 1.3, which is a must-know:

Frequency	Type	Wavelength
	—	
$10^{18} - 10^{20}$	$\gamma$ rays	$10^{-12} - 10^{-10}$
$10^{16} - 10^{18}$	X - rays	$10^{-10} - 10^{-8}$
$10^{15} - 10^{16}$	UV	$10^{-8} - 10^{-7}$
	_	
$10^{14} - 10^{15}$	blue	$10^{-7} - 10^{-6}$
$10^{14} - 10^{15}$	yellow	$10^{-7} - 10^{-6}$
$10^{14} - 10^{15}$	red	$10^{-7} - 10^{-6}$
	_	
$10^{11} - 10^{14}$	IR	$10^{-6} - 10^{-3}$
$10^9 - 10^{11}$	microwave	$10^{-3} - 10^{-1}$
$1 - 10^9$	radio	$10^{-1} - 10^8$

Observe the tiny space occupied by the visible light, all colors there, and the many more missing, being squeezed under the  $10^{14} - 10^{15}$  frequency banner. Here is a zoom on that part, with of course the remark that all this, colors, is something subjective:

Frequency $THz = 10^{12} Hz$	Color	Wavelength $nm = 10^{-9} m$
	_	
670 - 790	violet	380 - 450
620 - 670	blue	450 - 485
600 - 620	cyan	485 - 500
530 - 600	green	500 - 565
510 - 530	yellow	565 - 590
480 - 510	orange	590 - 625
400 - 480	red	625 - 750

Hang on, we are not done yet with the Maxwell equations, and their consequences. Yet another feature of these equations is that these can be regarded as well as a precursor of Einstein's relativity theory, which can be summarized as follows:

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FACT 1.4 (Relativity theory). The speeds are bounded, v < c, by the speed of light in vaccuum, which is the same for all intertial observers, given by:

 $c = 299\,792\,458 \text{ m/s}$ 

In view of this, classical mechanics must be fixed, and the correct formula for the addition of speeds, guaranteeing v < c for the sum, is Einstein's formula

$$v_{AC} = \frac{v_{AB} + v_{BC}}{1 + v_{AB}v_{BC}/c^2}$$

which at small speeds reduces to the usual Galileo formula  $v_{AC} = v_{AB} + v_{BC}$ . Moreover, the improved theory is invariant under the space-time Lorentz transformation

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t - vx/c^{2})$$

 $t' = \gamma(t - vx/c^2)$ where  $\gamma = 1/\sqrt{1 - v^2/c^2}$ , exactly as the Maxwell equations. Gravity can be added, too.

Obviously, many deep things going on here, and many other things can be said, for instance  $E = mc^2$  comes from this too. This being said, the idea of Einstein is very simple, based only on v < c. Indeed, by rescaling things as to have c = 1, we are looking for a speed addition formula  $(u, v) \rightarrow u + v$  satisfying the following condition:

$$u, v \leq 1 \implies u +_e v \leq 1$$

But here, thinking at the math, not many choices, with the obvious choice being:

$$u +_e v = \frac{u + v}{1 + uv}$$

And the miracle is that this formula, which is the one in the statement after rescaling by c, is indeed the correct one. With everything coming afterwards, namely Lorentz transformation, and gravity added, being more or less straightforward mathematics.

Finally, no discussion of relativity would be complete without a proof of  $E = mc^2$ . The idea here is that the relativistic energy of an object of rest mass m > 0 is as follows, making it clear that at speed v = 0, the energy should be  $E = mc^2$ :

$$\mathcal{E} = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$
$$= mc^2 \left(1 + \frac{v^2}{2c^2} + \dots\right)$$
$$= mc^2 + \frac{mv^2}{2} + \dots$$

#### 1. CELLS, LIFE

Now still speaking deep things, and going back to the Maxwell equations from Fact 1.3, although almighty, and compatible with relativity too, via the mathematics of the Lorentz transformation, these still do not solve the 2-body problem in electrodynamics, which is the functioning problem for the hydrogen atom. The problem comes from quantum mechanics, whose basic philosophy can be summarized as follows:

FACT 1.5 (Quantum mechanics). Small particles like electrons and protons do not have clear positions and speeds. This is how things are, at that scale, and it is all about the probability of finding the particle here or there, and with this or that speed.

This might seem overly vague, but sometimes a totally new and weird thought, of course in the hands of someone having the technical know-how, is enough to make science advance. Besides the above fact, which is something mathematical and theoretical, of key importance was the discovery, by Balmer, Rydberg and others, of the mechanism of the spectral lines of hydrogen H. These lines, depending on integer parameters  $n_1 < n_2$ , are given by the Rydberg formula, which is as follows, with R = 1.096 775 83 × 10<sup>7</sup>:

$$\frac{1}{\lambda_{n_1 n_2}} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Interestingly, and perhaps reminding a bit speed addition in relativity, these spectral lines combine according to the Ritz-Rydberg principle, which is as follows:

$$\frac{1}{\lambda_{n_1n_2}} + \frac{1}{\lambda_{n_2n_3}} = \frac{1}{\lambda_{n_1n_3}}$$

In practice, all these lines came from the efforts of several people, namely Balmer in 1885, in the visible range, then Lyman in 1906 in UV, Paschen in 1908 in IR, and later Brackett in 1922, Pfund in 1924, Humphreys in 1953, and others aftwerwards, with all the extra lines being in far IR. The simplified complete table is as follows:

$n_1$	$n_2$	Series name	Wavelength $n_2 = \infty$	Color $n_2 = \infty$
		—	—	
1	$2-\infty$	Lyman	91.13  nm	UV
2	$3-\infty$	Balmer	$364.51~\mathrm{nm}$	UV
3	$4-\infty$	Paschen	$820.14~\mathrm{nm}$	IR
		_	_	
4	$5-\infty$	Brackett	1458.03  nm	far IR
5	$6-\infty$	Pfund	2278.17  nm	far IR
6	$7-\infty$	Humphreys	$3280.56~\mathrm{nm}$	far IR
:				:
•	•		-	

Now back to the Ritz-Rydberg principle, which is the main theoretical result in all this, this reminds the following multiplication formula for the usual matrix units  $e_{ij}: e_j \to e_i$ ,

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perhaps taken in infinite dimensions, as to allow infinite-ranging indices:

$$e_{n_1 n_2} e_{n_2 n_3} = e_{n_1 n_3}$$

But this is very interesting, suggesting that the observables of the hydrogen atom should be some sort of infinite matrices, making the link with Fact 1.5.

Obviously, what we have here is a first-class scientific puzzle. Based on all this, and on some earlier predictions of Bohr, who was the initiator of the whole program, Heisenberg and Schrödinger, and then De Broglie, Dirac, Pauli and others were able to solve this puzzle, and develop a quantum mechanics theory starting from Fact 1.5, with the main applications, to the functioning of hydrogen and of other atoms, being as follows:

FACT 1.6 (Atomic theory). The atoms are formed by a core of protons and neutrons, surrounded by a cloud of electrons, basically obeying to a modified version of electromagnetism. And with a fine mechanism involved, as follows:

- (1) The electrons are free to move only on certain specified elliptic orbits, labelled  $1, 2, 3, \ldots$ , situated at certain specific heights.
- (2) The electrons can jump or fall between orbits  $n_1 < n_2$ , absorbing or emitting light and heat, that is, electromagnetic waves, as accelerating charges.
- (3) The energy of such a wave, coming from  $n_1 \rightarrow n_2$  or  $n_2 \rightarrow n_1$ , is given, via the Planck viewpoint, by the Rydberg formula, applied with  $n_1 < n_2$ .
- (4) The simplest such jumps are those observed by Lyman, Balmer, Paschen. And multiple jumps explain the Ritz-Rydberg formula.

Still with me, I hope? We are certainly now into complicated physics, and even seem to be somewhere towards the end of science, as understandable by humans. But, thinking well, we are in fact only at the beginning, because Fact 1.6 is not that useful as such, for the simple reason that atoms usually don't come alone, but rather tend to attach to each other, and form molecules. So, with physics understood, welcome to chemistry.

#### 1b. Into chemistry

Getting into chemistry now, we first need a better understanding of the atoms, as described by Fact 1.6. The basics of chemistry can be summarized as follows:

FACT 1.7 (Basic chemistry). Atoms can be labeled according to their atomic number, which is the number of protons in their nucleus, in practice

$$Z = 1, \dots, 118$$

and tend to attach to each other, and form molecules, with the electron distribution on the orbitals being responsible for this mechanism.

#### 1. CELLS, LIFE

All this is very interesting, and truly corresponding to what happens in the real life, meaning at our scale, our usual temperature, our usual pressure, and so on. More precisely now, there are two assertions here. First is a continuation of Fact 1.6, namely more atomic physics, which leads to the conclusion that the known atoms, also called chemical elements, basically depend only on their atomic number  $Z = 1, \ldots, 118$ . These chemical elements can be arranged in a table, called periodic table, as follows:

	1	2		3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	$\frac{\mathrm{H}}{\mathrm{1}}$																		$\frac{\text{He}}{2}$
2	$\frac{\text{Li}}{3}$	$\frac{\mathrm{Be}}{4}$												$\frac{\mathrm{B}}{5}$	$\frac{C}{6}$	$\frac{\mathrm{N}}{\mathrm{7}}$	$\frac{O}{8}$	$\frac{\mathrm{F}}{\mathrm{9}}$	$\frac{\mathrm{Ne}}{10}$
3	$\frac{\text{Na}}{11}$	$\frac{\mathrm{Mg}}{12}$												$\frac{\mathrm{Al}}{13}$	$\frac{\mathrm{Si}}{14}$	$\frac{P}{15}$	$\frac{S}{16}$	$\frac{\text{Cl}}{17}$	$\frac{\mathrm{Ar}}{\mathrm{18}}$
4	$\frac{\mathrm{K}}{\mathrm{19}}$	$\frac{\mathrm{Ca}}{20}$		$\frac{\mathrm{Sc}}{21}$	$\frac{\mathrm{Ti}}{22}$	$\frac{\mathrm{V}}{23}$	$\frac{\mathrm{Cr}}{24}$	$\frac{\mathrm{Mn}}{25}$	$\frac{\mathrm{Fe}}{26}$	$\frac{\mathrm{Co}}{27}$	$\frac{\mathrm{Ni}}{28}$	$\frac{\mathrm{Cu}}{29}$	$\frac{\mathrm{Zn}}{\mathrm{30}}$	$\frac{\text{Ga}}{31}$	$\frac{\mathrm{Ge}}{32}$	$\frac{\mathrm{As}}{33}$	$\frac{\mathrm{Se}}{34}$	$\frac{\mathrm{Br}}{35}$	$\frac{\mathrm{Kr}}{36}$
5	$\frac{\mathrm{Rb}}{37}$	$\frac{\mathrm{Sr}}{38}$		$\frac{Y}{39}$	$\frac{\mathrm{Zr}}{40}$	$\frac{\mathrm{Nb}}{41}$	$\frac{\mathrm{Mo}}{42}$	$\frac{\mathrm{Tc}}{43}$	$\frac{\mathrm{Ru}}{44}$	$\frac{\mathrm{Rh}}{45}$	$\frac{\mathrm{Pd}}{46}$	$\frac{\mathrm{Ag}}{47}$	$\frac{\mathrm{Cd}}{48}$	$\frac{\mathrm{In}}{49}$	$\frac{\mathrm{Sn}}{50}$	$\frac{\mathrm{Sb}}{51}$	$\frac{\text{Te}}{52}$	$\frac{1}{53}$	$\frac{Xe}{54}$
6	$\frac{\text{Cs}}{55}$	$\frac{\text{Ba}}{56}$	l	$\frac{\mathrm{Lu}}{71}$	$\frac{\mathrm{Hf}}{\mathrm{72}}$	$\frac{\mathrm{Ta}}{73}$	$\frac{W}{74}$	$\frac{\text{Re}}{75}$	$\frac{\mathrm{Os}}{76}$	$\frac{\mathrm{Ir}}{77}$	$\frac{\mathrm{Pt}}{78}$	$\frac{\mathrm{Au}}{79}$	$\frac{\mathrm{Hg}}{\mathrm{80}}$	$\frac{\mathrm{Tl}}{81}$	$\frac{\mathrm{Pb}}{\mathrm{82}}$	$\frac{\mathrm{Bi}}{83}$	$\frac{\mathrm{Po}}{\mathrm{84}}$	$\frac{\mathrm{At}}{85}$	$\frac{\mathrm{Rn}}{86}$
7	$\frac{\mathrm{Fr}}{87}$	$\frac{\text{Ra}}{88}$	a	$\frac{\mathrm{Lr}}{103}$	$\frac{\mathrm{Rf}}{104}$	$\frac{\mathrm{Db}}{\mathrm{105}}$	$\frac{Sg}{106}$	$\frac{\mathrm{Bh}}{107}$	$\frac{\mathrm{Hs}}{\mathrm{108}}$	$\frac{\mathrm{Mt}}{\mathrm{109}}$	$\frac{\mathrm{Ds}}{110}$	<u>Rg</u> 111	$\frac{\mathrm{Cn}}{112}$	$\frac{\mathrm{Nh}}{113}$	$\frac{\mathrm{Fl}}{114}$	$\frac{\mathrm{Mc}}{115}$	$\frac{Lv}{116}$	$\frac{\mathrm{Ts}}{117}$	<u>Og</u> 118
			l:	$\frac{\text{La}}{57}$	$\frac{\text{Ce}}{58}$	$\frac{\Pr}{59}$	$\frac{\mathrm{Nd}}{60}$	$\frac{\mathrm{Pm}}{61}$	$\frac{\mathrm{Sm}}{62}$	$\frac{\mathrm{Eu}}{63}$	$\frac{\mathrm{Gd}}{64}$	$\frac{\mathrm{Tb}}{65}$	$\frac{\mathrm{Dy}}{66}$	$\frac{\mathrm{Ho}}{67}$	$\frac{\mathrm{Er}}{68}$	$\frac{\mathrm{Tm}}{69}$	$\frac{\mathrm{Yb}}{70}$		
			a:	$\frac{Ac}{89}$	$\frac{\mathrm{Th}}{90}$	$\frac{\mathrm{Pa}}{91}$	$\frac{\mathrm{U}}{92}$	$\frac{\mathrm{Np}}{93}$	$\frac{\mathrm{Pu}}{94}$	$\frac{\mathrm{Am}}{95}$	$\frac{\mathrm{Cm}}{96}$	$\frac{\mathrm{Bk}}{97}$	$\frac{\mathrm{Cf}}{98}$	$\frac{\mathrm{Es}}{99}$	$\frac{\mathrm{Fm}}{100}$	$\frac{\mathrm{Md}}{\mathrm{101}}$	$\frac{\text{No}}{102}$		

Here the horizontal parameter  $1, \ldots, 18$  is called the group, and the vertical parameter  $1, \ldots, 7$  is called the period. The two rows on the bottom consist of lanthanum  ${}_{57}$ La and its followers, called lanthanides, and of actinium  ${}_{89}$ Ac and its followers, called actinides. These are to be inserted in the main table, where indicated, lanthanides between barium  ${}_{56}$ Ba and lutetium  ${}_{71}$ Lu, and actinides between radium  ${}_{88}$ Ra and lawrencium  ${}_{103}$ Lr.

Thus, the periodic table, when correctly drawn, but no one does that because of obvious typographical reasons, is in fact a  $7 \times 32$  table. Note here that, according to our  $7 \times 18$  convention, which is the standard one, lanthanides and actinides don't have a group number  $1, \ldots, 18$ . Their group is by definition "lanthanides" and "actinides".

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#### 1B. INTO CHEMISTRY

In order to go now towards chemistry, as a first requirement, you cannot call yourself a chemist if not knowing all the elements up to krypton  $_{36}$ Kr, which are absolutely needed for everything, at least a little bit. The names of these elements are as follows:

(1) Hydrogen  $_1$ H, helium  $_2$ He.

(2) Lithium <sub>3</sub>Li, berylium <sub>4</sub>Be, boron <sub>5</sub>B, carbon <sub>6</sub>C, nitrogen <sub>7</sub>N, oxygen <sub>8</sub>O, fluorine <sub>9</sub>F, neon <sub>10</sub>Ne.

(3) Sodium  $_{11}$ Na, magnesium  $_{12}$ Mg, aluminium  $_{13}$ Al, silicon  $_{14}$ Si, phosphorus  $_{15}$ P, sulfur  $_{16}$ S, chlorine  $_{17}$ Cl, argon  $_{18}$ Ar.

(4) Potassium  $_{19}$ K, calcium  $_{20}$ Ca, scandium  $_{21}$ Sc, titanium  $_{22}$ Ti, vanadium  $_{23}$ V, and chromium  $_{24}$ Cr, manganese  $_{25}$ Mn, iron  $_{26}$ Fe, cobalt  $_{27}$ Co.

(5) Nickel  $_{28}$ Ni, copper  $_{29}$ Cu, zinc  $_{30}$ Zn, gallium  $_{31}$ Ga, germanium  $_{32}$ Ge, arsenic  $_{33}$ As, selenium  $_{34}$ Se, bromine  $_{35}$ Br, krypton  $_{36}$ Kr.

Observe that all names fit with the abbreviations, expect for sodium  $_{11}$ Na, coming from the Latin natrium, potassium  $_{19}$ K, coming from the Latin kalium, iron  $_{26}$ Fe coming from the Latin ferrum, and also copper  $_{29}$ Cu, coming from the Latin cuprum.

In what regards the elements heavier than krypton  $_{36}$ Kr, it was heartbreaking to sort them out, we just love them all, but as a useful complement to the above list, we can at least list some remarkable elements, for various reasons, among them. These include:

- (6) Noble gases: xenon  ${}_{54}$ Xe, radon  ${}_{86}$ Rn.
- (7) Noble metals: silver  $_{47}$ Ag, iridium  $_{77}$ Ir, platinum  $_{78}$ Pt, gold  $_{47}$ Au.
- (8) Heavy metals: mercury  $_{80}$ Hg, lead  $_{82}$ Pb.
- (9) Radioactive: polonium  $_{84}$ Po, radium  $_{88}$ Ra, uranium  $_{92}$ U, plutonium  $_{94}$ Pu.

(10) Miscellaneous: rubidium  $_{37}$ Rb, strontium  $_{38}$ Sr, molybdenum  $_{42}$ Mo, technetium  $_{43}$ Tc, cadmium  $_{48}$ Cd, tin  $_{50}$ Sn, iodine  $_{53}$ I, caesium  $_{55}$ Cs, tungsten  $_{74}$ Tu, bismuth  $_{83}$ Bi, francium  $_{87}$ Fr, americium  $_{95}$ Am.

Here the abbreviations not fitting with English names come from the Latin or sometimes Greek argentum  $_{47}$ Ag, aurum  $_{47}$ Au, hydrargyrum  $_{80}$ Hg, plumbum  $_{82}$ Pb and stannum  $_{50}$ Sn. The noble gases in (1) normally include oganesson  $_{118}$ Og as well. The noble metals in (2) are something subjective. There are of course plenty of other heavy metals (3), or radioactive elements (4). As for the list in (5), this is something subjective, basically a

#### 1. CELLS, LIFE

mixture of well-known metals used in engineering, and some well-known bad guys in the context of nuclear fallout. Technetium  $_{43}$ Tc is a bizarre element, human-made.

Regarding now the second assertion in Fact 1.7, regarding the formation of molecules, this again comes from Fact 1.6, but via a more complicated mechanism. The idea here is that given two or several atoms, which can have the same atomic number Z or not, what happens is that, depending on their respective Z, these atoms might choose to share some electrons, with this coming somehow from less energy needed for functioning, in this new configuration. And so, we are led to clusters of atoms, called molecules.

As an example here, or rather counterexample, let us look at the group 18 elements, helium <sub>2</sub>He, neon <sub>10</sub>Ne, argon <sub>18</sub>Ar, krypton <sub>36</sub>Kr, xenon <sub>54</sub>Xe and radon <sub>86</sub>Rn. These are called noble gases, and are allergic to chemistry, because the group 18 elements are precisely those with all possible electron positions fully occupied, up to a certain  $n \in \mathbb{N}$ , which makes them very unfriendly to any chemistry proposition from the outside.

So long for the chemical elements, and the periodic table. Unfortunately business is business, and we will have to stop here, and go towards organic chemistry.

#### 1c. Organic molecules

Organic molecules.

#### 1d. Cells and life

Cells and life.

1e. Exercises

Exercises:

EXERCISE 1.8.

Exercise 1.9.

Exercise 1.10.

EXERCISE 1.11.

Exercise 1.12.

EXERCISE 1.13.

EXERCISE 1.14.

EXERCISE 1.15.

### Viruses, mutations

2a. Viruses

2b. Mutations

#### Viruses.

Mutations.

2c. Microbes

Microbes.

2d. Epidemiology

Epidemiology.

2e. Exercises

Exercises:

EXERCISE 2.1.

EXERCISE 2.2.

EXERCISE 2.3.

EXERCISE 2.4.

EXERCISE 2.5.

EXERCISE 2.6.

EXERCISE 2.7.

EXERCISE 2.8.

## Animals, vertebrates

3a. Fishes Fishes. **3b.** Animals Animals. 3c. Vertebrates Vertebrates. 3d. Humans Humans. 3e. Exercises Exercises: EXERCISE 3.1. EXERCISE 3.2. EXERCISE 3.3. EXERCISE 3.4. EXERCISE 3.5. EXERCISE 3.6. EXERCISE 3.7. EXERCISE 3.8. Bonus exercise.

## Apex predators

4a. A better world

A better world.

4b. Sharks

Sharks.

Crocodiles.

4d. Cats

4c. Crocodiles

Cats.

4e. Exercises

Exercises:

EXERCISE 4.1.

EXERCISE 4.2.

EXERCISE 4.3.

EXERCISE 4.4.

EXERCISE 4.5.

EXERCISE 4.6.

EXERCISE 4.7.

EXERCISE 4.8.

# Part II

# Fine mechanisms

Don't wanna hear about it Every single one's got a story to tell Everyone knows about it From the Queen of England to the Hounds of Hell

## Game theory

5a. Game theory

**5**b.

**5c.** 

5d.

#### 5e. Exercises

Exercises:

EXERCISE 5.1.

EXERCISE 5.2.

Exercise 5.3.

EXERCISE 5.4.

EXERCISE 5.5.

Exercise 5.6.

EXERCISE 5.7.

Exercise 5.8.

## Agriculture, famine

#### 6a. Agriculture, famine

6b.

6c.

6d.

#### 6e. Exercises

Exercises:

Exercise 6.1.

EXERCISE 6.2.

EXERCISE 6.3.

EXERCISE 6.4.

EXERCISE 6.5.

EXERCISE 6.6.

Exercise 6.7.

EXERCISE 6.8.

## Disease and drugs

#### 7a. Disease and drugs

7b.

7c.

#### 7d.

### 7e. Exercises

Exercises:

Exercise 7.1.

EXERCISE 7.2.

Exercise 7.3.

Exercise 7.4.

EXERCISE 7.5.

Exercise 7.6.

EXERCISE 7.7.

Exercise 7.8.

## Fighting, wars

8a. Fighting, wars

**8**b.

8c.

8d.

#### 8e. Exercises

Exercises:

Exercise 8.1.

Exercise 8.2.

Exercise 8.3.

EXERCISE 8.4.

EXERCISE 8.5.

EXERCISE 8.6.

EXERCISE 8.7.

EXERCISE 8.8.

# Part III

# Further examples

All I need is Co-ordination I can't imagine My destination

## Physics of matter

#### 9a. Physics of matter

9b.

9c.

9d.

#### 9e. Exercises

Exercises:

Exercise 9.1.

Exercise 9.2.

Exercise 9.3.

EXERCISE 9.4.

EXERCISE 9.5.

Exercise 9.6.

EXERCISE 9.7.

EXERCISE 9.8.

## Into the brain

10a. Into the brain

10b.

10c.

10d.

#### 10e. Exercises

Exercises:

Exercise 10.1.

Exercise 10.2.

Exercise 10.3.

EXERCISE 10.4.

EXERCISE 10.5.

EXERCISE 10.6.

EXERCISE 10.7.

EXERCISE 10.8.

## Computer science

#### 11a. Computer science

11b.

11c.

#### 11d.

#### 11e. Exercises

Exercises:

EXERCISE 11.1.

Exercise 11.2.

Exercise 11.3.

EXERCISE 11.4.

EXERCISE 11.5.

EXERCISE 11.6.

EXERCISE 11.7.

EXERCISE 11.8.

## Science of science

12a. Science of science

12b.

12c.

12d.

#### 12e. Exercises

Exercises:

EXERCISE 12.1.

Exercise 12.2.

Exercise 12.3.

EXERCISE 12.4.

EXERCISE 12.5.

EXERCISE 12.6.

EXERCISE 12.7.

EXERCISE 12.8.

Part IV

**Evolution denial** 

And still I stand this very day With a burning wish to fly away I'm still looking Looking for the summer

## Old religion

13a. Old religion

13b.

13c.

13d.

#### 13e. Exercises

Exercises:

Exercise 13.1.

Exercise 13.2.

Exercise 13.3.

EXERCISE 13.4.

EXERCISE 13.5.

Exercise 13.6.

EXERCISE 13.7.

EXERCISE 13.8.

## New religion

14a. New religion

14b.

14c.

14d.

#### 14e. Exercises

Exercises:

EXERCISE 14.1.

Exercise 14.2.

EXERCISE 14.3.

EXERCISE 14.4.

EXERCISE 14.5.

EXERCISE 14.6.

EXERCISE 14.7.

EXERCISE 14.8.

## Temples and priests

15a. Temples and priests

15b.

15c.

15d.

#### 15e. Exercises

Exercises:

Exercise 15.1.

Exercise 15.2.

Exercise 15.3.

Exercise 15.4.

Exercise 15.5.

Exercise 15.6.

EXERCISE 15.7.

EXERCISE 15.8.

## Towards darkness

16a. Towards darkness

16b.

16c.

16d.

16e. Exercises

Congratulations for having read this book, and no exercises for this final chapter.

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