Probability and statistics

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2010 *Mathematics Subject Classification.* 60C05

Key words and phrases. Probability, Statistics

Abstract. This is an introduction to probability and statistics. We first discuss the discrete random variables, with a general introduction to the subject, and the basic theory and examples, and with the material being focused on the binomial and Poisson laws, their combinatorics, and their various generalizations. We then make our way towards continuous probability, with some motivating examples, and some basic theory, and with the needed mathematical analysis tools explained too. As a continuation, we focus then on the normal variables, and their various versions and generalizations, with a detailed discussion about them. Finally, we provide an introduction to a number of more advanced aspects, mixing probability with combinatorics, geometry and physics.

Preface

This is an introduction to probability and statistics. We first discuss the discrete random variables, with a general introduction to the subject, and the basic theory and examples, and with the material being focused on the binomial and Poisson laws, their combinatorics, and their various generalizations. We then make our way towards continuous probability, with some motivating examples, and some basic theory, and with the needed mathematical analysis tools explained too. As a continuation, we focus then on the normal variables, and their various versions and generalizations, with a detailed discussion about them. Finally, we provide an introduction to a number of more advanced aspects, mixing probability with combinatorics, geometry and physics.

Cergy, April 2025
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Congratulations for having read this book, and no exercises for this final chapter.

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