

# The science of cooling

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ABSTRACT. This is an introduction to thermodynamics and thermal machines, and thermal engineering in general, with particular interest in various modern technologies, such as advanced cooling methods. We first discuss fire, how this appears, what its physics and chemistry are, and how this can be domesticated by man, and what simple machines can be built, with inspiration from it. We then go on a more advanced discussion, featuring controlled explosions and thermal engines, whose functioning and mathematics we discuss in some detail. Then we discuss cooling, freezing and related topics, again mostly with physics and engineering matters, and with a look as well at the various properties of the materials undergoing such transformations. Finally, we have a look at more modern technology, such as lasers, and the advanced cooling methods using them.

## Preface

This is an introduction to thermodynamics and thermal machines, and thermal engineering in general, with particular interest in various modern technologies, such as advanced cooling methods. We first discuss fire, how this appears, what its physics and chemistry are, and how this can be domesticated by man, and what simple machines can be built, with inspiration from it. We then go on a more advanced discussion, featuring controlled explosions and thermal engines, whose functioning and mathematics we discuss in some detail. Then we discuss cooling, freezing and related topics, again mostly with physics and engineering matters, and with a look as well at the various properties of the materials undergoing such transformations. Finally, we have a look at more modern technology, such as lasers, and the advanced cooling methods using them.

*Cergy, April 2025*

*Teo Banica*



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Part I

**Fire**

*And heaven can wait  
Cause you're my desire  
Heaven can wait  
You're turning up in time*

## CHAPTER 1

1a.

1b.

1c.

1d.

### 1e. Exercises

Exercises:

EXERCISE 1.1.

EXERCISE 1.2.

EXERCISE 1.3.

EXERCISE 1.4.

EXERCISE 1.5.

EXERCISE 1.6.

EXERCISE 1.7.

EXERCISE 1.8.

Bonus exercise.



## CHAPTER 2

**2a.**

**2b.**

**2c.**

**2d.**

### **2e. Exercises**

Exercises:

EXERCISE 2.1.

EXERCISE 2.2.

EXERCISE 2.3.

EXERCISE 2.4.

EXERCISE 2.5.

EXERCISE 2.6.

EXERCISE 2.7.

EXERCISE 2.8.

Bonus exercise.



## CHAPTER 3

**3a.**

**3b.**

**3c.**

**3d.**

### **3e. Exercises**

Exercises:

EXERCISE 3.1.

EXERCISE 3.2.

EXERCISE 3.3.

EXERCISE 3.4.

EXERCISE 3.5.

EXERCISE 3.6.

EXERCISE 3.7.

EXERCISE 3.8.

Bonus exercise.





## CHAPTER 4

4a.

4b.

4c.

4d.

### 4e. Exercises

Exercises:

EXERCISE 4.1.

EXERCISE 4.2.

EXERCISE 4.3.

EXERCISE 4.4.

EXERCISE 4.5.

EXERCISE 4.6.

EXERCISE 4.7.

EXERCISE 4.8.

Bonus exercise.



Part II

**Engines**

*Our brains are on fire  
With the feeling to kill  
And it won't go away  
Until our dreams are fulfilled*

## CHAPTER 5

5a.

5b.

5c.

5d.

### 5e. Exercises

Exercises:

EXERCISE 5.1.

EXERCISE 5.2.

EXERCISE 5.3.

EXERCISE 5.4.

EXERCISE 5.5.

EXERCISE 5.6.

EXERCISE 5.7.

EXERCISE 5.8.

Bonus exercise.



## CHAPTER 6

**6a.**

**6b.**

**6c.**

**6d.**

### **6e. Exercises**

Exercises:

EXERCISE 6.1.

EXERCISE 6.2.

EXERCISE 6.3.

EXERCISE 6.4.

EXERCISE 6.5.

EXERCISE 6.6.

EXERCISE 6.7.

EXERCISE 6.8.

Bonus exercise.





## CHAPTER 7

7a.

7b.

7c.

7d.

### 7e. Exercises

Exercises:

EXERCISE 7.1.

EXERCISE 7.2.

EXERCISE 7.3.

EXERCISE 7.4.

EXERCISE 7.5.

EXERCISE 7.6.

EXERCISE 7.7.

EXERCISE 7.8.

Bonus exercise.



## CHAPTER 8

8a.

8b.

8c.

8d.

### 8e. Exercises

Exercises:

EXERCISE 8.1.

EXERCISE 8.2.

EXERCISE 8.3.

EXERCISE 8.4.

EXERCISE 8.5.

EXERCISE 8.6.

EXERCISE 8.7.

EXERCISE 8.8.

Bonus exercise.



**Part III**  
**Cooling**

*This is the twilight zone  
And you're not on your own  
We'll take you to  
The twilight zone*

## CHAPTER 9

**9a.**

**9b.**

**9c.**

**9d.**

### **9e. Exercises**

Exercises:

EXERCISE 9.1.

EXERCISE 9.2.

EXERCISE 9.3.

EXERCISE 9.4.

EXERCISE 9.5.

EXERCISE 9.6.

EXERCISE 9.7.

EXERCISE 9.8.

Bonus exercise.





## CHAPTER 10

**10a.**

**10b.**

**10c.**

**10d.**

**10e. Exercises**

Exercises:

EXERCISE 10.1.

EXERCISE 10.2.

EXERCISE 10.3.

EXERCISE 10.4.

EXERCISE 10.5.

EXERCISE 10.6.

EXERCISE 10.7.

EXERCISE 10.8.

Bonus exercise.



## CHAPTER 11

**11a.**

**11b.**

**11c.**

**11d.**

**11e. Exercises**

Exercises:

EXERCISE 11.1.

EXERCISE 11.2.

EXERCISE 11.3.

EXERCISE 11.4.

EXERCISE 11.5.

EXERCISE 11.6.

EXERCISE 11.7.

EXERCISE 11.8.

Bonus exercise.



## CHAPTER 12

**12a.**

**12b.**

**12c.**

**12d.**

**12e. Exercises**

Exercises:

EXERCISE 12.1.

EXERCISE 12.2.

EXERCISE 12.3.

EXERCISE 12.4.

EXERCISE 12.5.

EXERCISE 12.6.

EXERCISE 12.7.

EXERCISE 12.8.

Bonus exercise.



**Part IV**

**Lasers**

*Boy and you just grown boy  
She said come over  
Come over  
She smiled at you, boy*



## CHAPTER 13

**13a.**

**13b.**

**13c.**

**13d.**

**13e. Exercises**

Exercises:

EXERCISE 13.1.

EXERCISE 13.2.

EXERCISE 13.3.

EXERCISE 13.4.

EXERCISE 13.5.

EXERCISE 13.6.

EXERCISE 13.7.

EXERCISE 13.8.

Bonus exercise.



## CHAPTER 14

14a.

14b.

14c.

14d.

14e. Exercises

Exercises:

EXERCISE 14.1.

EXERCISE 14.2.

EXERCISE 14.3.

EXERCISE 14.4.

EXERCISE 14.5.

EXERCISE 14.6.

EXERCISE 14.7.

EXERCISE 14.8.

Bonus exercise.



## CHAPTER 15

**15a.**

**15b.**

**15c.**

**15d.**

**15e. Exercises**

Exercises:

EXERCISE 15.1.

EXERCISE 15.2.

EXERCISE 15.3.

EXERCISE 15.4.

EXERCISE 15.5.

EXERCISE 15.6.

EXERCISE 15.7.

EXERCISE 15.8.

Bonus exercise.



## CHAPTER 16

**16a.**

**16b.**

**16c.**

**16d.**

**16e. Exercises**

Congratulations for having read this book, and no exercises for this final chapter.





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