

World of randomness

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ABSTRACT. This is a general introduction to probability, and its applications to science. We first discuss, motivated by mathematics, life and games, the basics of probability theory, notably with full explanations regarding the Poisson laws, and the normal laws. We then go on a more technical probability discussion, motivated by basic questions in physics, coming notably from thermodynamics, and statistical mechanics. As a continuation, we then go deeper into physics, and chemistry too, with a discussion of quantum mechanics and quantum chemistry, focusing on basic probabilistic aspects. Finally, we allow our atoms and molecules to produce life, biology, human beings and societies, and we end with an introduction to this, probability methods in social science.

Preface

This is a general introduction to probability, and its applications to science. We first discuss, motivated by mathematics, life and games, the basics of probability theory, notably with full explanations regarding the Poisson laws, and the normal laws. We then go on a more technical probability discussion, motivated by basic questions in physics, coming notably from thermodynamics, and statistical mechanics. As a continuation, we then go deeper into physics, and chemistry too, with a discussion of quantum mechanics and quantum chemistry, focusing on basic probabilistic aspects. Finally, we allow our atoms and molecules to produce life, biology, human beings and societies, and we end with an introduction to this, probability methods in social science.

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Part I

Games, probability

*I can't believe that it's real
The way that you make me feel
A burning deep down inside
A love that I cannot hide*

CHAPTER 1

1a.

1b.

1c.

1d.

1e. Exercises

Exercises:

EXERCISE 1.1.

EXERCISE 1.2.

EXERCISE 1.3.

EXERCISE 1.4.

EXERCISE 1.5.

EXERCISE 1.6.

EXERCISE 1.7.

EXERCISE 1.8.

Bonus exercise.

CHAPTER 2

2a.

2b.

2c.

2d.

2e. Exercises

Exercises:

EXERCISE 2.1.

EXERCISE 2.2.

EXERCISE 2.3.

EXERCISE 2.4.

EXERCISE 2.5.

EXERCISE 2.6.

EXERCISE 2.7.

EXERCISE 2.8.

Bonus exercise.

CHAPTER 3

3a.

3b.

3c.

3d.

3e. Exercises

Exercises:

EXERCISE 3.1.

EXERCISE 3.2.

EXERCISE 3.3.

EXERCISE 3.4.

EXERCISE 3.5.

EXERCISE 3.6.

EXERCISE 3.7.

EXERCISE 3.8.

Bonus exercise.

CHAPTER 4

4a.

4b.

4c.

4d.

4e. Exercises

Exercises:

EXERCISE 4.1.

EXERCISE 4.2.

EXERCISE 4.3.

EXERCISE 4.4.

EXERCISE 4.5.

EXERCISE 4.6.

EXERCISE 4.7.

EXERCISE 4.8.

Bonus exercise.

Part II

Physics, engineering

*Chiquitita, you and I cry
But the sun is still in the sky and shining above you
Let me hear you sing once more like you did before
Sing a new song, Chiquitita*

CHAPTER 5

5a.

5b.

5c.

5d.

5e. Exercises

Exercises:

EXERCISE 5.1.

EXERCISE 5.2.

EXERCISE 5.3.

EXERCISE 5.4.

EXERCISE 5.5.

EXERCISE 5.6.

EXERCISE 5.7.

EXERCISE 5.8.

Bonus exercise.

CHAPTER 6

6a.

6b.

6c.

6d.

6e. Exercises

Exercises:

EXERCISE 6.1.

EXERCISE 6.2.

EXERCISE 6.3.

EXERCISE 6.4.

EXERCISE 6.5.

EXERCISE 6.6.

EXERCISE 6.7.

EXERCISE 6.8.

Bonus exercise.

CHAPTER 7

7a.

7b.

7c.

7d.

7e. Exercises

Exercises:

EXERCISE 7.1.

EXERCISE 7.2.

EXERCISE 7.3.

EXERCISE 7.4.

EXERCISE 7.5.

EXERCISE 7.6.

EXERCISE 7.7.

EXERCISE 7.8.

Bonus exercise.

CHAPTER 8

8a.

8b.

8c.

8d.

8e. Exercises

Exercises:

EXERCISE 8.1.

EXERCISE 8.2.

EXERCISE 8.3.

EXERCISE 8.4.

EXERCISE 8.5.

EXERCISE 8.6.

EXERCISE 8.7.

EXERCISE 8.8.

Bonus exercise.

Part III

Matter, chemistry

*If you think you're getting away
I will prove you wrong
I'll take you all the way
Boy, just come along*

CHAPTER 9

9a.

9b.

9c.

9d.

9e. Exercises

Exercises:

EXERCISE 9.1.

EXERCISE 9.2.

EXERCISE 9.3.

EXERCISE 9.4.

EXERCISE 9.5.

EXERCISE 9.6.

EXERCISE 9.7.

EXERCISE 9.8.

Bonus exercise.

CHAPTER 10

10a.

10b.

10c.

10d.

10e. Exercises

Exercises:

EXERCISE 10.1.

EXERCISE 10.2.

EXERCISE 10.3.

EXERCISE 10.4.

EXERCISE 10.5.

EXERCISE 10.6.

EXERCISE 10.7.

EXERCISE 10.8.

Bonus exercise.

CHAPTER 11

11a.

11b.

11c.

11d.

11e. Exercises

Exercises:

EXERCISE 11.1.

EXERCISE 11.2.

EXERCISE 11.3.

EXERCISE 11.4.

EXERCISE 11.5.

EXERCISE 11.6.

EXERCISE 11.7.

EXERCISE 11.8.

Bonus exercise.

CHAPTER 12

12a.

12b.

12c.

12d.

12e. Exercises

Exercises:

EXERCISE 12.1.

EXERCISE 12.2.

EXERCISE 12.3.

EXERCISE 12.4.

EXERCISE 12.5.

EXERCISE 12.6.

EXERCISE 12.7.

EXERCISE 12.8.

Bonus exercise.

Part IV

Social science

*I got the eye of the tiger, a fighter
Dancing through the fire
Cause I am a champion
And you're gonna hear me roar*

CHAPTER 13

13a.

13b.

13c.

13d.

13e. Exercises

Exercises:

EXERCISE 13.1.

EXERCISE 13.2.

EXERCISE 13.3.

EXERCISE 13.4.

EXERCISE 13.5.

EXERCISE 13.6.

EXERCISE 13.7.

EXERCISE 13.8.

Bonus exercise.

CHAPTER 14

14a.

14b.

14c.

14d.

14e. Exercises

Exercises:

EXERCISE 14.1.

EXERCISE 14.2.

EXERCISE 14.3.

EXERCISE 14.4.

EXERCISE 14.5.

EXERCISE 14.6.

EXERCISE 14.7.

EXERCISE 14.8.

Bonus exercise.

CHAPTER 15

15a.

15b.

15c.

15d.

15e. Exercises

Exercises:

EXERCISE 15.1.

EXERCISE 15.2.

EXERCISE 15.3.

EXERCISE 15.4.

EXERCISE 15.5.

EXERCISE 15.6.

EXERCISE 15.7.

EXERCISE 15.8.

Bonus exercise.

CHAPTER 16

16a.

16b.

16c.

16d.

16e. Exercises

Congratulations for having read this book, and no exercises for this final chapter.

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